## HEAT ENGINES

Two diagrams show the difference between a heat engine (device to produce heat from work) that satisfies the $1^{\text {st }}$ Law BUT DOES NOT satisfy the $2^{\text {nd }}$ Law (a) and one that satisfieS both the $1^{\text {st }}$ and $2^{\text {nd }}$ Laws (b)
(a)


Heat Engine

$$
\mathbf{Q}_{\mathrm{in}}=\mathbf{W}_{\text {net }}
$$

$$
\eta=1 \text { (by only } 1^{\text {st }} \text { Law) }
$$

## $x$ Violates Kelvin-Planck statement of $2^{\text {nd }}$ Law.

(b)


Equivalent formulas for efficiency of a heat engine

$$
\eta=\frac{W_{\text {net }}}{\mathrm{Q}_{\text {net }}}=\frac{\mathrm{w}_{\text {net }}}{\mathrm{q}_{\mathrm{H}}}=\frac{\dot{\mathrm{W}}_{\text {net }}}{\dot{\mathrm{Q}}_{\mathrm{H}}}=1-\frac{\mathrm{Q}_{\mathrm{L}}}{\mathrm{Q}_{\mathrm{H}}}=1-\frac{\mathrm{q}_{\mathrm{L}}}{\mathrm{q}_{\mathrm{H}}}=1-\frac{\dot{\mathrm{Q}}_{\mathrm{L}}}{\dot{\mathrm{Q}}_{\mathrm{H}}}
$$

Solving heat engine efficiency problems:

1. given $\eta$, find $W_{\text {net }}$ or $Q_{H}$, by finding either $W$ or $Q$ from the $1^{\text {st }} \mathrm{Law}$ and use efficiency formula to find the unknown (W or Q ).
2. given W find Q , or given Q find W from $1^{\text {st }}$ Law and use formula to calculate $\eta$

## REFRIGERATORS

Two diagrams show the difference between a heat engine (device to produce heat from work) that satisfies the $1^{\text {st }}$ Law BUT DOES NOT satisfy the $2^{\text {nd }}$ Law (a) and one that satisfieS both the $1^{\text {st }}$ and $2^{\text {nd }}$ Laws (b)

$\checkmark$ Satisfies $1^{\text {st }}$ Law:
$\mathbf{Q}_{\mathbf{H}}=\mathbf{Q}_{\mathbf{L}}$
$\mathrm{COP}_{\mathrm{R}}$ undefined (div 0)
$x$ Violates Clausius’
statement of $2^{\text {nd }}$ Law.
(b)

$\checkmark$ Satisfies $1^{\text {st }}$ Law:
$\mathbf{W}_{\text {in }}=\mathbf{Q}_{\mathrm{L}}-\mathbf{Q}_{\mathrm{H}}$
$\mathrm{COP}_{\mathrm{R}}=\mathrm{Q}_{\mathrm{L}} / \mathrm{W}_{\text {in }}$
$\checkmark$ Satisfies $2^{\text {nd }}$ Law since work input, $\mathrm{W}_{\text {in }}>0$
*Heat pump is same as refrigerator EXCEPT
$\mathbf{C O P}_{\mathrm{HP}}=\mathbf{Q}_{\mathrm{H}} / \mathbf{W}_{\text {in }}$

Equivalent formulas for $\mathrm{COP}_{\mathrm{R}}$ and $\mathrm{COP}_{\mathrm{HP}}$
REFRIGERATOR
$\mathrm{COP}_{\mathrm{R}}=\frac{\mathrm{Q}_{\mathrm{L}}}{\mathrm{W}_{\text {in }}}=\frac{\dot{\mathrm{Q}}_{\mathrm{L}}}{\dot{\mathrm{W}}_{\text {in }}}=\frac{\mathrm{q}_{\mathrm{L}}}{\mathrm{W}_{\text {in }}}=\frac{1}{\left(\frac{\mathrm{Q}_{\mathrm{H}}}{\mathrm{Q}_{\mathrm{L}}}-1\right)}=\frac{1}{\left(\frac{\dot{\mathrm{Q}}_{\mathrm{H}}}{\dot{\mathrm{Q}}_{\mathrm{L}}}-1\right)}=\frac{1}{\left(\frac{\mathrm{q}_{\mathrm{H}}}{\mathrm{q}_{\mathrm{L}}}-1\right)}$

## HEAT PUMP

$$
\mathrm{COP}_{\mathrm{HP}}=\frac{\mathrm{Q}_{\mathrm{H}}}{\mathrm{~W}_{\mathrm{in}}}=\frac{\dot{\mathrm{Q}}_{\mathrm{H}}}{\dot{\mathrm{~W}}_{\mathrm{in}}}=\frac{\mathrm{q}_{\mathrm{H}}}{\mathrm{~W}_{\text {in }}}=\frac{1}{\left(1-\frac{\mathrm{Q}_{\mathrm{L}}}{\mathrm{Q}_{\mathrm{H}}}\right)}=\frac{1}{\left(1-\frac{\dot{\mathrm{Q}}_{\mathrm{L}}}{\dot{\mathrm{Q}}_{\mathrm{H}}}\right)}=\frac{1}{\left(1-\frac{\mathrm{q}_{\mathrm{L}}}{\mathrm{q}_{\mathrm{H}}}\right)}
$$

NOTE THAT:
Coefficient of Performance values are generally > 1
Also, it can be shown that:

$$
\mathbf{C O P}_{\mathrm{HP}}=\mathbf{C O P}_{\mathrm{R}}+1
$$

Since $\operatorname{COP}_{R}+1=\left(\frac{Q_{L}}{Q_{H}-Q_{L}}\right)+1=\frac{Q_{H}}{Q_{H}-Q_{L}}=\operatorname{COP}_{H P}$

$$
\mathbf{A N D}\left|\mathbf{Q}_{\mathbf{L}}\right|<\left|\mathbf{Q}_{\mathbf{H}}\right|
$$

Since $\mathrm{W}_{\mathrm{in}}=\mathrm{Q}_{\mathrm{L}}-\mathrm{Q}_{\mathrm{H}}\left(1^{\text {st }}\right.$ law) and $\mathrm{W}_{\mathrm{in}}$ is negative, then $\left(\mathrm{Q}_{\mathrm{L}}-\mathrm{Q}_{\mathrm{H}}\right)$ must be negative.

Solving refrigeration and heat pump COP problems:

1. Given COP, find W , or $\mathrm{Q}_{\mathrm{L}} / \mathrm{Q}_{\mathrm{H}}$ using $1^{\text {st }}$ Law, and use COP formula to find second (unknown) $\mathrm{Q}_{\mathrm{L}} / \mathrm{Q}_{\mathrm{H}}$ or W .
2. Given $W$ or $Q_{L} / Q_{H}$, find unknown $W$ or $Q_{L} / Q_{H}$ from $1^{\text {st }}$ Law and calculate COP.
