MODELING OF CONCRETE MATERIALS AND STRUCTURES

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Class Meeting #8: Micromorphic Continuum Formulations

Higher Grade Continua:

Include Gradient, Micropolar and Nonlocal Formulations

Internal Length Scale:

Regularizes Softening Formulations Independently of Mesh Size

Micromorphic Finite Elements:

Increase Stiffness Properties when compared to Classical Finite Displacement Elements

REGULARIZATION OF ILL-POSEDNESS

Mesh Objectivity of Localized Failure Simulations:

- Higher Grade Continua:
  - Gradient Formulations
  - Cosserat Formulations
  - Micromorphic Formulations
- Nonlocal Methods:
  - Nonlocal Kinematic Measures
  - Nonlocal Kinetic Measures
  - Nonlocal Internal Descriptions
- Kinematic Discontinuities:
  - PUM Methods
  - Multiple Interfaces with Fixed Orientation
  - Adaptive Crack Tracking methods

HIGHER GRADE CONTINUUM FE MODEL

Micromorphic Continua: Mindlin [1963], Eringen [1999]

Micromorphic Two Field Finite Element Expansion: $\boldsymbol{u}(\boldsymbol{x}, t)$ and $\boldsymbol{h}(\boldsymbol{x}, t)$

1. Nodal Displacements (2-D): $\boldsymbol{u}_I = [u_1, u_2]^t$

2. Nodal Micro-Deformations (2-D): $\boldsymbol{h}_I = [h_{11}, h_{22}, h_{12}, h_{21}]^t$

(a) Macro Deformations: symmetric

$$\boldsymbol{\epsilon}(\boldsymbol{x}, t) = \nabla_x^{sym} \boldsymbol{N}_I^u(\boldsymbol{x}) \boldsymbol{u}_I(t)$$

(b) Relative Micro-Deformations: non-symmetric

$$\boldsymbol{e}(\boldsymbol{x}, t) = \nabla_x \boldsymbol{N}_I^u(\boldsymbol{x}) \boldsymbol{u}_I(t) - \bar{\boldsymbol{N}}_I^h(\boldsymbol{x}) \boldsymbol{h}_I(t)$$

(c) Micro Curvatures = Micro Deformation Gradient: non-symmetric

$$\boldsymbol{\kappa}(\boldsymbol{x}, t) = \nabla_x \bar{\boldsymbol{N}}_I^h(\boldsymbol{x}) \boldsymbol{h}_I(t)$$

MICROMORPHIC FINITE ELEMENT

Conjugate Stresses for Linear Isotropic Elastic Behavior:

(a) Macro Stress-Deformation Relationship: Symmetric $\boldsymbol{\sigma} = \boldsymbol{E}^m : \boldsymbol{\epsilon}$

$$\boldsymbol{E}^m = K^m \mathbf{1} \otimes \mathbf{1} + 2G^m \boldsymbol{I}^{s,dev}$$

(b) Relative Stress-Deformation Relationship: Non-Symmetric $\boldsymbol{s} = \boldsymbol{E}^{rel} : \boldsymbol{e}$

$$\boldsymbol{E}^{rel} = K^{rel} \mathbf{1} \otimes \mathbf{1} + 2G^{rel} \boldsymbol{I}^{s,dev} + 2G_c^{rel} \boldsymbol{I}^{a,dev}$$

(c) Micro Moment-Curvature Relationship: Non-symmetric $\boldsymbol{\mu} = \boldsymbol{E}^{\mu} : \cdot \boldsymbol{\kappa}$

$$\boldsymbol{E}^{\mu} = 2\ell_c^2 \boldsymbol{G}^*$$

Note characteristic length scale ℓ^2 due membrane-bending interaction

MICROMORPHIC FINITE ELEMENT

MM Element Stiffness Partitions:

$$\left[\begin{array}{c|c} \boldsymbol{k}_m^{uu} + \boldsymbol{k}_{rel}^{uu} & \boldsymbol{k}_{rel}^{uh} \\ \hline \boldsymbol{k}_{rel}^{hu} & \boldsymbol{k}_{\mu}^{hh} + \boldsymbol{k}_{rel}^{hh} \end{array}\right]_{32\times32} \left[\begin{array}{c} \boldsymbol{u} \\ \hline \boldsymbol{h} \end{array}\right]_{32\times1} = \left[\begin{array}{c} \boldsymbol{f}^u \\ \hline \boldsymbol{f}^h \end{array}\right]^{ext}_{32\times1}$$

(a) Macroscopic Stiffness: Standard displacement partition $\delta W_m^{int} = \int \delta\boldsymbol{\epsilon} : \boldsymbol{\sigma} dV$

$$\boldsymbol{k}_m^{uu} = \int_V [\boldsymbol{B}^u]^t[\boldsymbol{E}^m][\boldsymbol{B}^u] dV$$

(b) Relative Micro Stiffness:‘membrane-bending’ interaction $\delta W_{rel}^{int} = \int \delta\boldsymbol{e} : \boldsymbol{s} dV$

$$\boldsymbol{k}_{rel}^{uh} = \int_V [\bar{\boldsymbol{B}}^u - \bar{\boldsymbol{N}}^h]^t[\boldsymbol{E}^{rel}][\bar{\boldsymbol{B}}^u - \bar{\boldsymbol{N}}^h] dV$$

(c) Micro-Curvature Stiffness: introduces length-scale $\delta W_{\mu}^{int} = \int \delta\boldsymbol{\kappa} : \boldsymbol{\mu} dV$

$$\boldsymbol{k}_{\mu}^{hh} = \int_V [\bar{\boldsymbol{B}}^h]^t[\boldsymbol{E}^{\mu}][\bar{\boldsymbol{B}}^h] dV$$

DIFFERENCE OF MACRO AND MICROMORPHIC STIFFNESS

Difference between macro and micromorphic stiffness properties:
$\boldsymbol{k}_{dif} = \boldsymbol{k}_{mm} - \boldsymbol{k}_{m}$

(a) MM Schur Complement:

$$[\boldsymbol{k}_{mm}^{uu}]_{16\times16} = [\boldsymbol{k}_{m}^{uu} + \boldsymbol{k}_{rel}^{uu}]_{16\times16} - \left[\boldsymbol{k}_{rel}^{uh}\right]_{16\times16} \left[\boldsymbol{k}_{\mu}^{hh} + \boldsymbol{k}_{rel}^{hh}\right]_{16\times16}^{-1} \left[\boldsymbol{k}_{rel}^{hu}\right]_{16\times16}$$

(b) Standard Macro Element:

$$\boldsymbol{k}_{m}^{uu} = \int_V [\boldsymbol{B}^u]^t [\boldsymbol{E}^m][\boldsymbol{B}^u] dV$$

Stiffness difference is positive semi-definite:

$$\lambda_i(\boldsymbol{k}_{dif}) \geq 0 \quad \text{such that} \quad \boldsymbol{k}_{mm} \geq \boldsymbol{k}_{m}$$

Micromorphic stiffness is larger than or at least equal to the macroscopic stiffness (equality holds for rigid body and constant energy states).

MULTIPLE REPOSITORIES OF INTERNAL ENERGIES

Difference of macro and micromorphic strain energy contributions:

(a) Standard Macro Element:

$$W_{macro} = W^{\sigma}_{sym} = \frac{1}{2}\{\boldsymbol{u}^t_I \, \boldsymbol{k}^{uu}_m \; \boldsymbol{u}_J\} = \frac{1}{2}\{\boldsymbol{u}^t_I[\int_V [\boldsymbol{B}^u]^t[\boldsymbol{E}^m][\boldsymbol{B}^u]dV]\,\boldsymbol{u}_J\}$$

(b) Micromorphic Element:

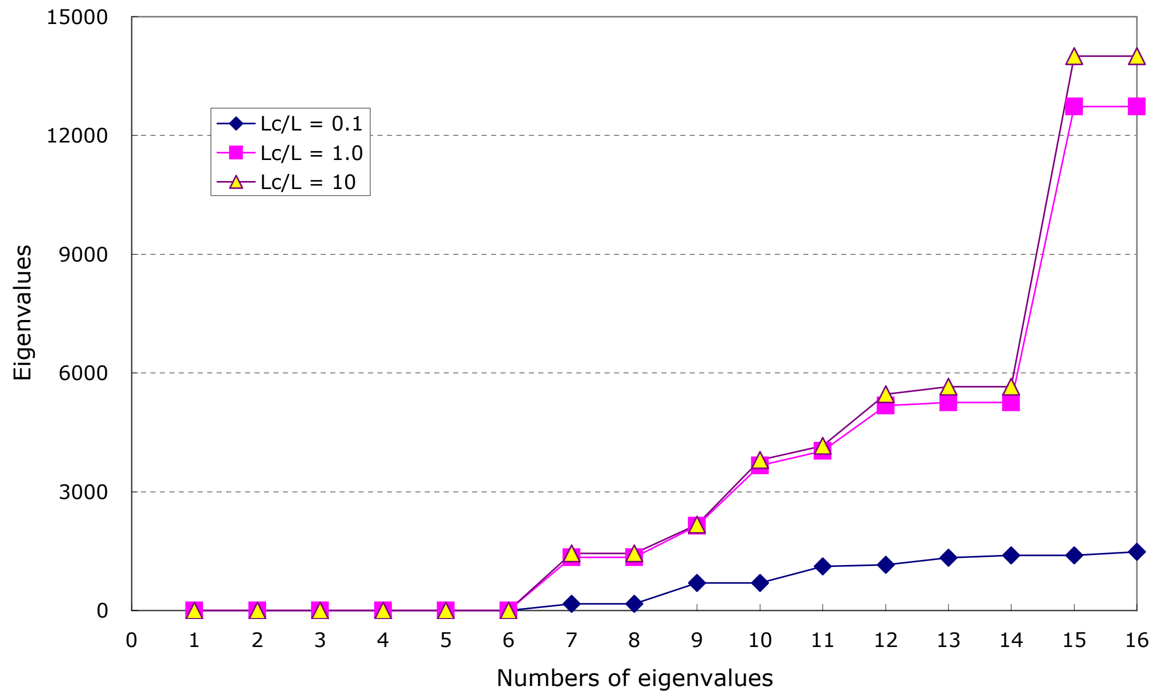
$$W_{m-morphic} = W^{\sigma}_{sym} + W^{s}_{sym} + W^{s}_{skew} + W^{\mu}_{sym} + W^{\mu}_{skew} = \frac{1}{2}\{\boldsymbol{d}^t_I \, \boldsymbol{k}_{mm} \; \boldsymbol{d}_J\}$$

Both formulations coincide for RBM and constant energy states whereby $\boldsymbol{d}_I = [\boldsymbol{u}_I, \boldsymbol{h}_I]$.

STIFFNESS DIFFERENCE

Effect of Internal Length Scale $\ell_c - \ell_e$:

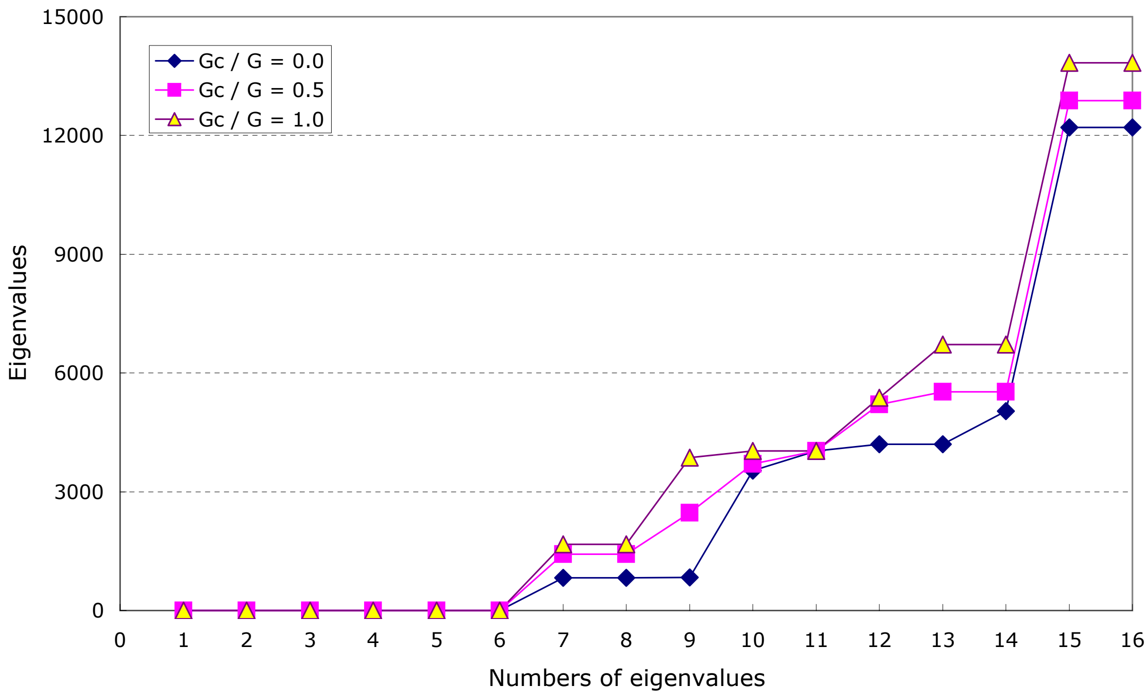
Eigenvalues of Difference Matrix: $\lambda_i(\boldsymbol{k}_{mm} - \boldsymbol{k}_m) \geq 0$



STIFFNESS DIFFERENCE

Effect of anti-symmetry in $\boldsymbol{s} - \boldsymbol{e}$:

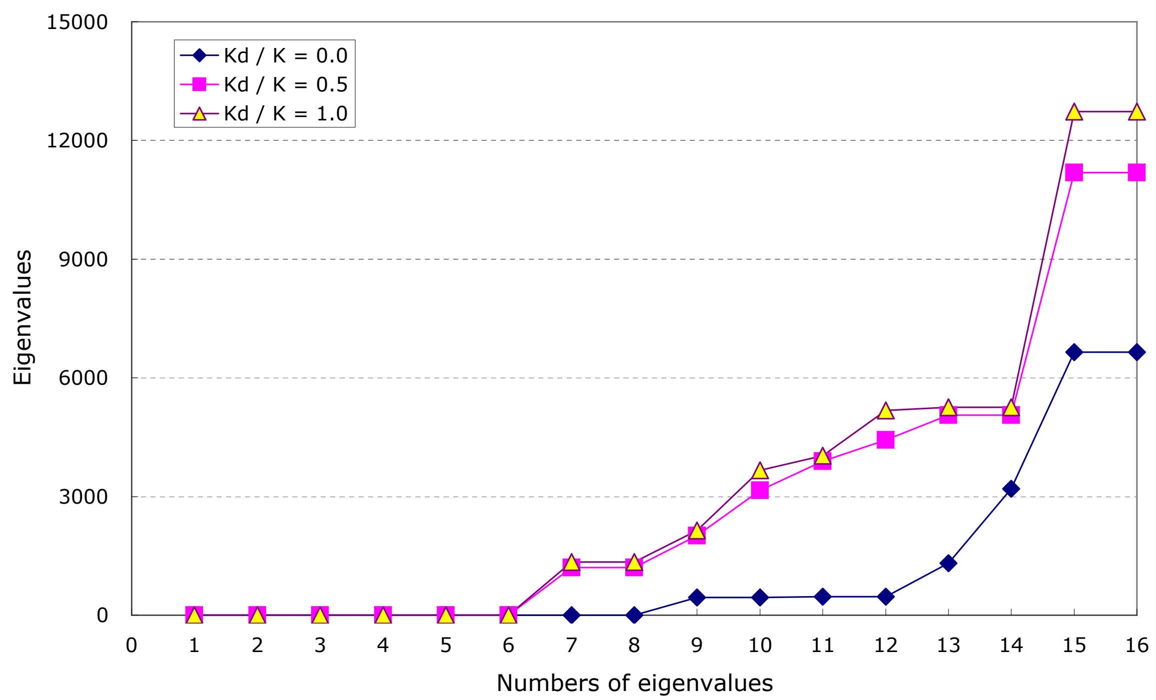
Eigenvalues of Stiffness Difference Matrix: $\lambda_i(\boldsymbol{k}_{mm} - \boldsymbol{k}_m) \geq 0$



STIFFNESS DIFFERENCE

Effect of Bulk Damage: $K_d = [1 - d_K] K_o$:

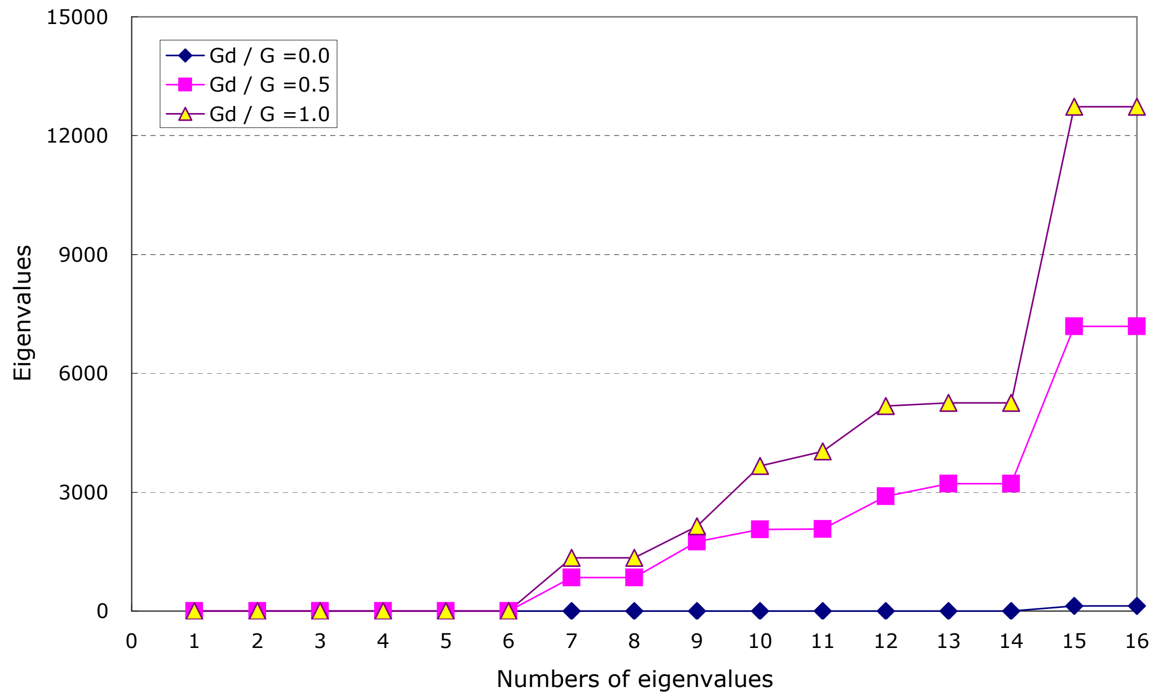
Eigenvalues of Stiffness Difference Matrix: $\lambda_i(\boldsymbol{k}_{mm} - \boldsymbol{k}_m) \geq 0$



STIFFNESS DIFFERENCE

Effect of Shear Damage: $G_d = [1 - d_G]G_o$:

Eigenvalues of Stiffness Difference Matrix: $\lambda_i(\boldsymbol{k}_{mm} - \boldsymbol{k}_m) \geq 0$



CONCLUDING REMARKS

Main Lessons from Class # 8:

Length Scale:

Effect of Multiscale Interactions.

Stiffening Properties:

Effect of Multiple Energy Contributions.

MM-FE Discretization:

Static Condensation of Degrees of Freedom for Micro-Deformations ?