CVEN 6525 - Spring 2002

Finite Element Analysis of Structures

Homework # 1: 1-dim Finite Element

1. Problem:
   Consider the axial deformation problem of the bar with a circular cross-section. Develop the element properties which govern the axial motion $\mathbf{m}\,\ddot{\mathbf{u}}+\mathbf{k}\,\mathbf{u} = \mathbf{f}_p+\mathbf{f}_o$ of a three node bar element based on quadratic displacement expansion.

   Determine symbolic expressions for the stiffness and mass matrices, $\mathbf{k}, \mathbf{m}$ as well as the consistent and initial load vectors, $\mathbf{f}_p, \mathbf{f}_o$. Evaluate the mode shapes and frequencies of the quadratic bar element.

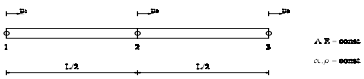

Figure 1: 1-D Bar Problem: Quadratic Bar Element with three Nodes

2. Problem:
   Assuming linearly elastic material behavior, determine the stress, strain, displacement and axial force field when the circular bar is subjected to the three load cases
   - a concentrated end load, P
   - a uniformly distributed load, $p = const$
   - a linear temperature distribution, $\Delta T(x) = \frac{x}{L}(T - T_o)$.

   Determine numerically the stiffness and mass properties as well as the internal nodal forces, $\mathbf{f}^{int} = \mathbf{k}\,\mathbf{u}$ and the external nodal forces $\mathbf{f}^{ext} = \mathbf{f}_p$ and the initial loads $\mathbf{f}^{initl} = \mathbf{f}_o$. Plot the mode shapes and solve the bar problem assuming node 1 to be pinned. Plot the finite element results in the form of diagrams of the axial normal force, the stress, the axial displacement and the strain, $N(x), \sigma(x), u(x), \epsilon(x)$ and determine their error when compared to the analytical solution.

   Material data for steel : $E = 30,000\,ksi$, $\alpha = 12x10^{-6}/C^o$, $\rho = 15.2 slugs/ft^3$
   Geometrical data: $L = 30\,in$, $d = 2\,in$
   Load data: $P = 80\,kips$, $p = 3\,kips/in$, and $\Delta T_{tip} = 30\,C^o$.