CVEN 6525 - Spring 2002

Finite Element Analysis of Structures

Homework # 2: Two-dim QUAD9 Element

Problem # 1:

Develop a plane stress/plane strain finite element stiffness properties based on bi-quadratic expansion of both the geometry as well as the displacement field using the isoparametric FE concept. Construct the (18×18) stiffness matrix $\mathbf{k}_e = \int \mathbf{B}^t\mathbf{E}\mathbf{B}\mathbf{dV}$ for isotropic elasticity ($E = 30,000\,ksi$, $\nu = 0.3$) with the help of numerical integration using (3×3) Gaussian quadrature for the unit square and the distorted element geometry below.

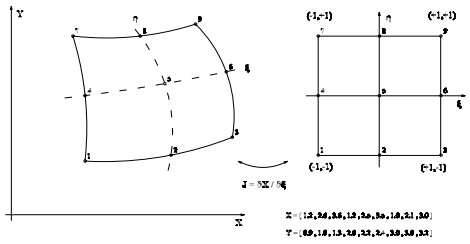

Figure 1: 2-D Plane Stress Problem: Bi-Quadratic Finite Element with nine Nodes

Problem # 2:

Assess the validity of your stiffness matrix by checking the rigid body mode requirement (zero column sum) for the unit square and the distorted element geometry. Determine the eigenvalues of the element when $\nu = 0.0, 0.3, 0.499$ for plane stress and plane strain stiffness properties and discuss the difference of the elastic stiffness properties if you reduce the order of integration from (3×3) to (2×2) and (1×1).