1. Air enters a compressor at 120 kPa and 40 °C and velocity of 10 m/s with an inlet area of 100 cm². The compressor heat loss per unit mass is 50 kJ/kg.

![Compressor Diagram]

a) What is the mass flow rate (in kg/s)?

\[
\dot{m} = \frac{\rho}{
\left(\frac{\rho}{RT}\right)_1
\]

\[
\rho = \frac{P_1}{RT_1} = \frac{120 \text{kPa}}{0.287 \text{kJ/kg \cdot K} \cdot (273 + 40) \text{K}} = \frac{1.336 \text{ kg}}{\text{m}^3}
\]

\[
\dot{m} = \frac{1.336 \text{ kg}}{\text{s}} \cdot \frac{10 \text{ m}}{\text{s}} \cdot \frac{0.01 \text{ m}^2}{\text{m}^2} = \frac{0.1336 \text{ kg}}{\text{s}}
\]

b) What is the required input power to compress the gas to 700 kPa and 160 °C (in kW)? You can assume that kinetic (and potential) energy changes are negligible compared with other energy changes.

\[
Q - W = \Delta U = \dot{m} C_p (T_2 - T_1), \quad \dot{Q} = \dot{m} q
\]

\[
W = \dot{m} (C_p (T_2 - T_1) - q)
\]

\[
= \frac{0.1336 \text{ kg}}{\text{s}} \cdot \frac{1.010 \text{ kJ/kg \cdot K}}{\text{k}} \cdot (160 - 40) \text{K} \cdot (-50 \text{ kJ/kg})
\]

\[
W = -22.9 \text{ kW}
\]
2. A house has a volume of 50 m³. The house contains many appliances which draw 1.2 kW of electricity on average and shelters 3 people who produce about 150W each. Atmospheric pressure is 100 kPa.

(a) If the house gains heat from the outdoors at a rate of 700W, how much cooling power must be provided to keep the temperature at a constant 70 °F? (Unfortunately, thermostat reads only in °F.)

\[
\dot{Q} - \dot{W} = \Delta U = 0
\]

\[
\dot{Q} = \dot{W}
\]

\[
0.7\text{kw} + 0.45\text{kw} = -1.2\text{kw}
\]

\[
\frac{\dot{Q}_c}{-2.35\text{kw}}
\]

(b) If the house were sealed to eliminate any heat loss or heat gain from the outside, how long would it take for the house to heat up by 10 °F with no cooling?

\[
1^\circ F = 1.8^\circ C
\]

\[
\Delta T = 10^\circ F = 10 = 5.56^\circ C
\]

\[
\frac{5.56^\circ C}{1.8} = 5.46^{\circ}K
\]

\[
\frac{5.56^{\circ}K}{2.73 + 2.11} = 21.1^{\circ}C
\]

\[
M = \frac{100 \cdot 3 \cdot (50 \text{m}^3)}{0.287\text{kJ}} (273 + 21.1)
\]

\[
= 59.2\text{kg}
\]

\[
\frac{(1.2\text{kw} + 0.45\text{kw})\Delta t}{mc\Delta T} = \frac{mCv\Delta T}{1.65\text{kJ/s}}
\]

\[
\Delta t = \frac{59.2\text{kg} \cdot (0.718\text{kJ/kg}) \cdot (5.56\text{K})}{1.65\text{kJ/s}} = \frac{143.5}{60\text{sec/min}} = 2.4\text{min}
\]

(c) What is the net work of the process in part b?

[1.2 kW]

(only electricity is work)
3. Steam flowing at 25 kg/s, 10 MPa, and 400 °C is expanded in an adiabatic turbine. At the turbine outlet the density of the wet steam is 0.728 kg/m³ and the pressure is 100 kPa.

a) Find the temperature of the steam at the turbine outlet

\[ T = T_{sat} \mid_{100 \text{ kPa}} = 99.63^\circ \text{C} \]

b) Find the moisture content (% liquid) of the steam at the turbine outlet

\[ X = \frac{v_2 - v_f}{v_{fg}} \mid_{100 \text{ kPa}} = \frac{1.3736 - 0.001043}{1.6940 - 0.001043} = 0.811\]

\[ X = \frac{m_g}{m_T} \]

\[ 1 - X = 1 - \frac{m_g}{m_T} = \frac{m_f}{m_T} \]

\[ = 1 - 0.811 = 0.189 \]

\[ \% \text{ moisture} = 18.9\% \]

c) Find the power output of the turbine

\[ \dot{Q} - \dot{W} = \dot{m} \Delta h_{i} \]

\[ \dot{W} = 25 \frac{\text{kg}}{\text{s}} \left( h_2 - h_1 \right) \]

\[ h_1 = 3096.5 \frac{\text{kJ}}{\text{kg}} \]

\[ h_2 = h_f + X h_{fg} \mid_{100 \text{ kPa}} \]

\[ = 417.46 + 0.811(2258.0) \]

\[ = 2248.7 \frac{\text{kJ}}{\text{kg}} \]

\[ \dot{W} = 25 \frac{\text{kg}}{\text{s}} \left( 2248.7 - 3096.5 \right) \]

\[ \dot{W} = -21,125 \text{ kW} \]

\[ \dot{W} = 21.2 \text{ MW} \]
d) The wet steam is reheated after it leaves the turbine to a pressure of 8 MPa and 400 °C. How much heat is required?

\[ Q - W = \dot{m} (h_3 - h_2) \]

\[ h_3 = 3138.3 \text{ kJ/kg} \]

\[ Q = \frac{25}{8} \left( -2248.7 \right) = 22240 \text{ kW} = 22.2 \text{ MW} \]

e) Draw the turbine and reheat processes on the T-v diagram below showing values for temperature and specific volume for each equilibrium state.

\[ \nu_1 = 0.024 \]
\[ \nu_2 = 1.336 \]
\[ \nu_3 = 0.02935 \]
4. Refrigerant (R-134a) is condensed from saturated vapor to saturated liquid at 1 MPa in a heat exchanger. After condensation, the saturated liquid is throttled in an adiabatic device to obtain a liquid-vapor mixture at 0.14 MPa.

a) Show the process of condensation and throttling on the T-v diagram below including equilibrium temperatures at the beginning and end of the two-step process. (Don’t forget to show process direction.)

![T-v diagram with points 1, 2, 3 and temperatures 39.39°C and -18.8°C]

b) What fraction of the refrigerant mass is liquid after throttling?

\[
\begin{align*}
\text{throttling - assume ideal isenthalpic} \\
\dot{h}_3 &= \dot{h}_2 = \dot{h}_f \bigg|_{1 MPa} = 105.29 \text{ kJ/kg} \\
X &= \frac{\dot{h}_3 - \dot{h}_f}{\dot{h}_f} \bigg|_{0.14 MPa} = \frac{105.29 - 25.77}{210.27} = 0.378
\end{align*}
\]

\[
\begin{align*}
\dot{m}_{f} &= 1 - X = 1 - \frac{\dot{m}_g}{\dot{m}_T} = \dot{m}_T - \dot{m}_g = \frac{\dot{m}_f}{\dot{m}_T} \\
\dot{m}_T \dot{m}_L &= 1 - 0.378 = \boxed{0.622}
\end{align*}
\]
c) What is the net energy transfer during the two-step process.

\[ q_{\text{net}} = h_3 - h_1 = h_2 - h_1 = h_{cg} \text{limp} \]

\[ q_{\text{net}} = 162.68 \text{ kJ/kg} \]

5. A rigid-walled cylinder is divided in half by a plate that does not move. The cylinder is well insulated.

![Diagram of a cylinder divided into two sections with H₂ and O₂, and an arrow indicating very good insulation.]

Given initial conditions:
- Hydrogen gas (H₂): T = 100 °C, P = 600 kPa, V = 1 m³
- Oxygen gas (O₂): T = 20 °C, P = 400 kPa, V = 1 m³

After a time, thermal equilibrium between the two compartments is reached. No mass is transferred across the plate, only heat.

a. Write the first law statement for the process and state all assumptions.

\[ \Delta U = 0 = m_H \cdot C_{v,H} (\Delta T)_H + m_O \cdot C_{v,O} (\Delta T)_O \]
b. Can you assume hydrogen and oxygen are ideal gases? Justify your assumption.

\[
\begin{align*}
\text{Yes} & \quad T_{e, H} = 33.3 \text{K} \ll 373 \text{K}, \quad P_{e, H} = 1.3 \text{MPa} \gg 600 \text{kPa} \\
T_{e, O} = 154.8 \text{K} \ll 293 \text{K}, \quad P_{e, O} = 5.08 \text{MPa} \gg 400 \text{kPa}
\end{align*}
\]

c. Find the equilibrium temperature. Assume that the specific heat values at room temperature (300 K) are okay to use.

\[
\begin{align*}
M_0 &= \frac{400 \text{kPa} \text{ (m}^3\text{)}}{0.2598 \text{kJ/kg}(273+20) \text{K}} = 5.25 \text{kg} \\
M_H &= \frac{600 \text{kPa} \text{ (m}^3\text{)}}{2.0769 \text{kJ/kg}(273+20) \text{K}} = 0.775 \text{kg}
\end{align*}
\]

\[
0.775 \text{kJ/kg} \left(100 - T_e\right) = 5.25 \text{kg} \left(0.658 \text{kJ/kg}\right)\left(T_e - 20\right)
\]

\[
2.41.3 - 2.41T_e = 3.45T_e - 69.1
\]

\[
310.4 = 5.86T_e
\]

\[
\begin{align*}
\left| T_e = 53^\circ \text{C} \right|
\end{align*}
\]

d. Find the final pressure in both compartments after thermal equilibrium is reached.

\[
P_{2, H} = \frac{0.2598 \text{kJ/kg}(273+53)(5.25)}{1} = 444.6 \text{ kPa}
\]

\[
P_{2, O} = \frac{2.0769 \text{kJ/kg}(273+53)(0.775)}{1} = 524.7 \text{ kPa}
\]
6. Hot water is cooled by air flow in a well-insulated heat exchanger. The hot water inlet temperature is 80 °C, and the outlet temperature is 30 °C. Air flows to the air inlet at 800 m³/min at a pressure of 100 kPa and a temperature of 27 °C. The exit air pressure is 95 kPa and temperature = 60 °C.

a) Find the mass flow rate of water that satisfies the conditions given.

\[ \dot{Q} = \dot{m}_w c_{pw}(30 - 80) + \dot{m}_a c_{pa}(60 - 27) \]

\[ \dot{m}_w = \frac{15.5 \text{ kg} \cdot 1.005 \text{ kJ/kgK} (33) \text{ K}}{4.184 \text{ kJ/kgK} (50) \text{ K}} = 1.45 \text{ kg/s} \]

\[ \rho_i = \frac{P_i}{RT_i} = \frac{100}{0.287(300)} = 1.16 \text{ kg/m}^3 \]

\[ \dot{m}_a = \frac{800 \text{ m}^3}{60 \text{ s}} \cdot \frac{1.16 \text{ kg}}{\text{m}^3} = 15.5 \text{ kg/s} \]

\[ c_{pa} = 1.005 \text{ kJ/kgK} \]

b) Find the volumetric air flow rate at the air outlet.

\[ \dot{V}_{a,e} = \frac{\dot{m}_a}{\rho_e} = \frac{15.5}{(95/0.287)(273+60)} = 15.6 \text{ m}^3/\text{s} \]

\[ = 936 \text{ m}^3/\text{min} \]
7. The Ranque-Hilsch vortex tube is a device that produces a low-temperature gas stream that is used for cooling manufactured parts. The device is well insulated, has no external work entering or leaving the system, and has no moving parts. For this particular device, air flows in at a mass flow rate of 2 kg/s, 4 atmospheres pressure and 20 °C (293 K). The air stream is split into two equal exit streams of 1 kg/s each, both at 1 atmosphere pressure, the cooling stream temperature is -20 °C (253 K).

a) What is the temperature of the exit air at point 2?

\[
\dot{Q} - \dot{W} = \dot{m}_2 h_2 + \dot{m}_3 h_3 - \dot{m}_1 h_1.
\]

\[
\dot{Q} = \dot{m}_2 h_2 + \dot{m}_3 h_3 - (\dot{m}_2 + \dot{m}_3) h_1
\]

Assume \( C_p \) same, and since \( \dot{m}_2 = \dot{m}_3 = 1 \) kg/s

\[
T_1 - T_2 = T_3 - T_1
\]

\[
20 - T_2 = -20 - 20
\]

\[
\boxed{T_2 = 60^\circ C}
\]