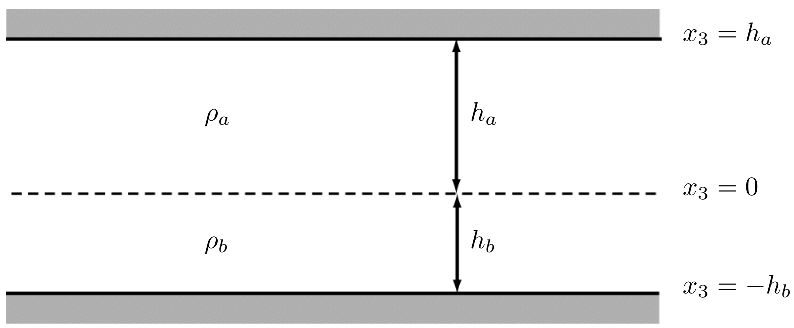
CVEN6833: Advanced Environmental Fluid Mechanics

Problem Set #1: Waves

1. Consider a two-layer stratified system as shown in the figure. Waves propagate on the interface of the two fluids (at $x_3 = 0$).



   (a) Show that the dispersion relation and phase speed are given by the expressions below:

$$\omega^2 = gk \frac{(\rho_b - \rho_a)}{\rho_a \coth kh_a + \rho_b \coth kh_b} \tag{1}$$

$$c_p^2 = \frac{g}{k} \frac{(\rho_b - \rho_a)}{\rho_a \coth kh_a + \rho_b \coth kh_b} \tag{2}$$

   (b) Determine expressions for the velocity fields in each of the layers.

2. Consider particle paths in deep-water waves. In particular, I would like you to investigate the sensitivity of your results to the "order" (i.e. number of terms) of your solution.

   (a) Calculate the stokes drift velocity (both horizontal and vertical) analytically for the zeroth, first, and second-order cases. Repeat for even higher-order cases until you can establish a pattern or trend (i.e. do higher-order terms keep contributing to the solution?). Explain why the vertical stokes velocity is identically zero (from a physical perspective).

   (b) Now repeat this excercise, but do it by numerically integrating Lagrangian particle paths for the various orders. Check your code by making sure that the zero-order solution produces a perfectly closed orbit. Plot the particle orbits for each case.

   (c) Using results presented in class, show that the first-order Stokes drift velocity for waves in a finite-depth H is

$$\bar{u}_L = a^2 \omega k \frac{cosh 2k(z_0 + H)}{2 \sinh^2 kH} \tag{3}$$