CVEN6833: Advanced Environmental Fluid Mechanics

Problem Set #2: Correlations and Spectra

1. (Based on Kundu, Problem 13.2) A periodic velocity field is given by $u(t) = \bar{U} + U_0 \cos \omega t$
   (a) Calculate the mean and standard deviation of $u(t)$.
   (b) Calculate the autocorrelation function $\overline{u(t)u(t+\tau)}$. Explain why your result is periodic.

2. (Based on Kundu, Problem 13.4) Calcuate $R_{12}(0) = \overline{u_1(t)u_2(t)}$ for the velocity records $u_1 = \cos \omega t$ and $u_2 = \cos(\omega t + \phi)$.
   (a) Plot $R_{12}(0)$ vs. ϕ
   (b) Make scatter diagrams (see Fig. 13.6 in Kundu) of u_2 vs u_1 for $\phi = 0$, $\pi/4$, and $\pi/2$.
   (c) Based on your scatter plots, argue that a straight line signifies perfect correlation, an ellipse a partial correlatioon, and a circle zero correlation.

3. Download the LDA velocity time-history from the website (right-click on the link and save to your computer). The data consists of approximately 100,000 streamwise velocity values sampled and recorded at 68.2 Hz.
   (a) Calculate the mean, rms, and variance of the velocity data in the time domain (i.e. without using any spectral methods).
   (b) Write a code to implement a FFT-based approach to calculating the PSD of the data, as discussed in class. The only built-in function you should use is the FFT. Implement the capability to calculate the spectra using multiple FFT blocks with windowing (i.e. Hanning) and block overlap.
   (c) Investigate the effect of varying the window function (i.e. boxcar window - which is really no window - vs. Hanning).
   (d) Investigate the effect of varying the block size
   (e) Integrate your spectrum to calculate the variance in the spectral domain. Make sure you get the same answer that you got in the time domain!

4. Using the same streamwise data from the previous question, calculate the following:
   (a) The autocorrelation function
   (b) The integral timescale

5. I will also put on the website vertical velocities corresponding to the horizontal velocities. As discussed in class, produce an animation of the location of the velocity records in the u' - w' plane.