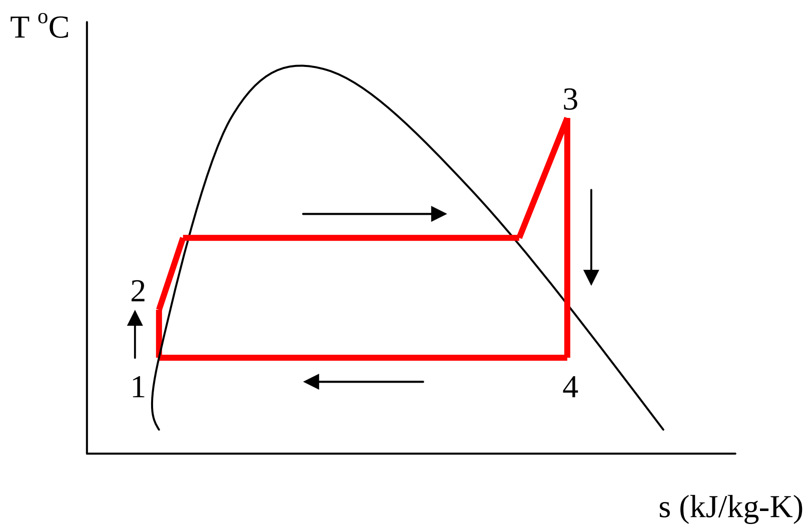
RANKINE POWER GENERATION CYCLE

A HEAT ENGINE: PRODUCES WORK FROM HEAT BY WASTING A FRACTION OF HEAT INPUT TO A LOW TEMPERATURE RESERVOIR



CHARACTERISTICS

1. Rankine cycle is a heat engine comprised of four internally reversible processes.

Significance: area enclosed by process lines equals the net heat transfer by Clausius' Principle for internally reversible processes:

$$dQ = TdS$$

for a cycle:

$$Q_{net} = \oint TdS$$

and by the First Law for a cycle: $Q_{net} = W_{net}$

$$W_{turbine} + W_{pump} = Q_H + Q_L$$

where $W_{turbine}$ is positive, W_{pump} is negative, Q_H is positive, Q_L is negative, $|Q_H| > |Q_L|$ and $|W_{turbine}| > |W_{pump}|$.

2. Process analysis using the First Law:

	Process Phases	Device	1^{st} Law expression
1→2	adiabatic and reversible (isentropic) compression input: w_{pump} Saturated Liquid→ Compressed liquid	pump	$-w_{pump} = (h_2 – h_1) = v(P_2 – P_1)$
2→3	isobaric expansion q_H = heat input from high temp reservoir Liquid → Superheated (usually) vapor	boiler	$q_H = (h_3 – h_2)$
3→4	adiabatic and reversible (isentropic) expansion output: $w_{turbine} >> w_{pump}$ Superheated vapor → Liquid-vapor mixture (usually)	turbine	$-w_{turbine} = (h_4 – h_3)$
4→1	isothermal and isobaric compression q_L = heat rejected to low temp reservoir Liquid-vapor mixture→ Saturated liquid.	condenser	$q_L = (h_1 – h_4)$

Note from the 1^{st} Law expressions that in the two processes that are adiabatic and reversible (isentropic), the pump and the turbine, the only energy interaction with surroundings is work. For the two isobaric heat exchange processes, the devices are passive and there is no work, only heat transfer.

3. Heat transfer in the ideal Rankine Cycle relies on phase change, a very efficient way to store and release energy. The working fluid is usually water/steam. During the cycle, the properties of the working fluid change as below with associated heat/work exchanges.

	h (kJ/kg)	P (kPa)	T (°C)	s (kJ/kg-k)	q or w (kJ/kg)
1→2	Small increase $h_1 = h_f$ @ P_1 $h_2 = h_1 + v(P_2 - P_1)$	Large Increase $P_2 >> P_1$	Negligible change $T_2 \approx T_1$ $T_1 = T_s$ @P_1	Constant $s_1 = s_f$ @ P_1 $s_2 = s_1$	$W_{pump} < 0$ $-w_{pump} = (h_2 - h_1)$ $h_2 > h_1$
2→3	Large increase $h_3 = h$ @ (P_3, T_3)	Constant $P_3 = P_2$	Increase $T_3 > T_2$	Increase $s_3 = s$ @ (P_3, T_3)	$q_H > 0$ $q_H = (h_3 - h_2)$ $h_3 > h_2$
3→4	Decrease $h_4 = x_4(h_{fg}) + h_f$ @ (P_4, T_4)	Large Decrease $P_4 < P_3$	Large Decrease $T_4 = T_s$ @P_4	Constant $s_3 = s_4$ $x_4 = (s_4 - s_f)/s_{fg}$	$W_{turbine} > 0$ $-w_{turbine} = (h_4 - h_3)$ $h_4 < h_3$
4→1	Large Decrease $h_1 < h_4$	Constant $P_4 = P_1$	Constant $T_4 = T_1 = T_s$ @ $P_{1,4}$	Large Decrease $s_1 < s_4$	$q_L < 0$ $q_L = (h_1 - h_4)$ $h_1 < h_4$

4. Analysis:

Efficiency:

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{w_{net}}{q_H} = \frac{[(h_3 - h_4) - (h_2 - h_1)]}{(h_3 - h_2)} = 1 - \frac{\dot{Q}_L}{\dot{Q}_H} = 1 - \frac{q_L}{q_H} = 1 - \frac{(h_4 - h_1)}{(h_3 - h_2)}$$

Also, the mass flow rate can be calculated using these relations:

$$\dot{m} = \frac{\dot{Q}_H}{q_H} = \frac{\dot{Q}_H}{(h_3 - h_2)} = \frac{\dot{Q}_L}{q_L} = \frac{\dot{Q}_L}{(h_4 - h_1)} = \frac{\dot{W}_{net}}{\eta(h_3 - h_2)}$$

REMINDER: FOR CALCULATING η, USE ABSOLUTE VALUES FOR HEAT AND WORK TERMS.

5. ENTROPY GENERATION IN RANKINE CYCLES

For the cycle:

$$\Delta s = 0$$

then

$$s_{gen} = -\sum_k \left(\frac{q}{T}\right)_k = -\frac{q_H}{T_{HTR}} - \frac{q_L}{T_{LTR}} = -\frac{(h_3 - h_2)}{T_{HTR}} - \frac{(h_1 - h_4)}{T_{LTR}} \quad \left(\frac{kJ}{kg - K}\right)$$

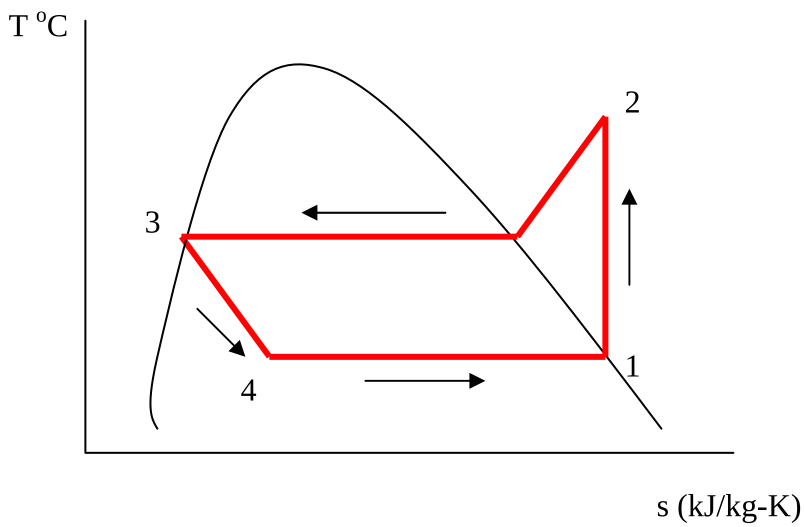
and

$$\dot{S}_{gen} = \dot{m} s_{gen} \quad \left(\frac{kw}{K}\right)$$

where $q_H > 0$, $q_L < 0$, and although $|q_L| < |q_H|$, $T_{HTR} >> T_{LTR}$ assuring that $-\frac{q_L}{T_{LTR}}(\text{positive}) > -\frac{q_H}{T_{HTR}}(\text{negative})$ and $s_{gen} > 0$, making Rankine Cycle "possible."

VAPOR-COMPRESSION REFRIGERATION (VCR) CYCLE FOR REFRIGERATORS AND HEAT PUMPS

VCR TRANSFERS HEAT FROM A LOW TEMPERATURE RESERVOIR TO A HIGH TEMPERATURE RESERVOIR WITH WORK INPUT.



CHARACTERISTICS

1. VCR cycle is comprised of three internally reversible processes.

One process is actually irreversible; however, the area enclosed by the process lines can be considered to approximately indicate the net heat transfer, and the First Law for a cycle applies:

$$Q_{net} = W = Q_H + Q_L$$

where W is negative, Q_H is negative, Q_L is positive, and $|Q_H| > |Q_L|$.

2. Process analysis using the First Law:

	Process Phases	Device	1^{st} Law expression
1→2	adiabatic and reversible (isentropic) compression input: $w_{compressor}$ Saturated Vapor→ Superheated Vapor	compressor	$-w_{compressor} = (h_2 - h_1)$
2→3	isobaric compression q_H = heat rejected to high temp reservoir Superheated vapor → saturated liquid	condenser	$q_H = (h_3 - h_2)$
3→4	adiabatic expansion (<u>not isentropic</u>: $s_3 \neq s_4$) Saturated Liquid → Liquid-vapor mixture	throttling valve	$h_4 = h_3$
4→1	isothermal and isobaric expansion q_L = heat input from low temp reservoir Liquid-vapor mixture→ Saturated vapor	evaporator	$q_L = (h_1 - h_4)$

Note from the 1^{st} Law expressions that in the one process that is adiabatic and reversible (isentropic), the compressor, the only energy interaction with surroundings is work. For the two isobaric heat exchange processes, the devices are passive and there is no work, only heat transfer. The adiabatic throttling valve is isenthalpic, NOT isentropic, as can be seen from the T-s diagram.

3. Heat transfer in the ideal VCR Cycle relies on phase change, a very efficient way to store and release energy. The working fluid is usually refrigerant (e.g. R-134a), a compound that boils (evaporates) at very low temperatures at near-atmospheric pressures. During the cycle, the properties of the working fluid change as below with associated heat/work exchanges.

	h (kJ/kg)	P (kPa)	T (^oC)	s (kJ/kg-k)	q or w (kJ/kg)
1→2	Increase $h_1 = h_g$ @ P_1 $h_2 = h$ @ (P_2,T_2)	Large Increase $P_2 >> P_1$	Increase $T_2 = T$@ (P_2, s_1) $T_1 = T_s$ @P_1	Constant $s_1 = s_g$ @ P_1 $s_2 = s_1$	$W_{compressor} < 0$ $-w_{compressor} = (h_2-h_1)$ $h_2 > h_1$
2→3	Large decrease $h_3 = h_f$ @ P_3	Constant $P_3 = P_2$	Decrease $T_2 > T_3$	Decrease $s_3 = s_f$ @ (P_3,T_3)	$q_H < 0$ $q_H = (h_3-h_2)$ $h_3 < h_2$
3→4	Constant $h_4 = h_3$ $x_4 = (h_4-h_f)/h_{fg}$ @ P_4	Large Decrease $P_4 < P_3$	Decrease $T_4 = T_s$ @P_4	Increase $s_3 = x_4(s_{fg})+ s_f$ @P_4	Both 0
4→1	Large increase $h_1 > h_4$	Constant $P_4 = P_1$	Constant $T_4 = T_1 = T_s$ @ $P_{1,4}$	Large increase $s_1 > s_4$	$q_L > 0$ $q_L = (h_1-h_4)$ $h_1 > h_4$

4. Analysis:

In general, the measure of VCR cycle performance is:

$$\text{Coefficient of Performance (COP)} = \frac{\text{Desired heat transfer}}{\text{Required work input}}$$

For a **refrigerator (air conditioner)** desired heat transfer is cooling - transfer of heat to the evaporator from the low temperature reservoir.

$$COP_R = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{q_L}{w_{in}} = \frac{(h_1 - h_4)}{(h_2 - h_1)} = \frac{1}{\left(\frac{\dot{Q}_H}{\dot{Q}_L} - 1\right)} = \frac{1}{\left(\frac{q_H}{q_L} - 1\right)} = \frac{1}{\left(\frac{(h_2 - h_3)}{(h_1 - h_4)} - 1\right)}$$

For a **heat pump** desired heat transfer is heating - transfer of heat from the condenser to the high temperature reservoir.

$$COP_{HP} = \frac{\dot{Q}_H}{\dot{W}_{in}} = \frac{q_H}{w_{in}} = \frac{(h_2 - h_3)}{(h_2 - h_1)} = \frac{1}{\left(1 - \frac{\dot{Q}_L}{\dot{Q}_H}\right)} = \frac{1}{\left(1 - \frac{q_L}{q_H}\right)} = \frac{1}{\left(1 - \frac{(h_1 - h_4)}{(h_2 - h_3)}\right)}$$

$COP_{HP} = COP_R + 1$ and therefore, $|q_H| > |q_L|$

Also the mass flow rate in the VCR cycle can be calculated using the relations:

$$\dot{m} = \frac{\dot{Q}_H}{q_H} = \frac{\dot{Q}_H}{(h_3 - h_2)} = \frac{\dot{Q}_L}{q_L} = \frac{\dot{Q}_L}{(h_4 - h_1)} = \frac{\dot{Q}_L}{COP_R (h_2 - h_1)} = \frac{\dot{Q}_H}{COP_{HP} (h_2 - h_1)}$$

REMINDER: FOR CALCULATING COP_R AND COP_{HP}, USE ABSOLUTE VALUES FOR HEAT AND WORK TERMS.

ENTROPY GENERATION IN VCR CYCLES

For the cycle:

$$\Delta s = 0$$

then

$$s_{gen} = -\sum_{k}\left(\frac{q}{T}\right)_{k} = -\frac{q_H}{T_{HTR}} - \frac{q_L}{T_{LTR}} = -\frac{(h_3 - h_2)}{T_{HTR}} - \frac{(h_1 - h_4)}{T_{LTR}} \quad \left(\frac{kJ}{kg-K}\right)$$

and

$$\dot{S}_{gen} = \dot{m}s_{gen} \quad \left(\frac{kw}{K}\right)$$

where $q_H < 0$, $q_L > 0$, and $|q_L| < |q_H|$. Also, T_{HTR} is close to T_{LTR} assuring that $-\frac{q_H}{T_{HTR}}(\text{positive}) > -\frac{q_L}{T_{LTR}}(\text{negative})$ and $s_{gen} > 0$, making VCR cycle "possible."