

AREN 2110 Second Law: Entropy, and Cycle Formula Summary Sheet

ENTROPY BALANCES

I. CLOSED SYSTEM

A. **Irreversible** processes:

$$\Delta S = S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + S_{gen} \left(\frac{kJ}{K} \right)$$

B. **Reversible** processes: $S_{gen} = 0$

$$S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} \left(\frac{kJ}{K} \right)$$

C. **Adiabatic irreversible** processes: $Q = 0$

$$S_2 - S_1 = S_{gen} \left(\frac{kJ}{K} \right)$$

D. **Adiabatic and reversible** processes

$$S_2 - S_1 = 0$$

E. **Cycle**

$$0 = \oint \frac{\delta Q}{T} + S_{gen} \left(\frac{kJ}{K} \right) = \sum_k \frac{Q_k}{T_{surr}} + S_{gen} \left(\frac{kJ}{K} \right)$$

II. OPEN SYSTEM (CONTROL VOLUME)

A. **Non-steady-state** and **irreversible** control volume

$$\frac{dS_{cv}}{dt} = \sum_k \left(\frac{\dot{Q}}{T} \right)_k + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{S}_{gen} \left(\frac{kw}{K} \right)$$

subscript i is for inlet and e is for outlet

B. **Steady-State** and irreversible control volume

$$\sum_e \dot{m}_e s_e - \sum_i \dot{m}_i s_i = \sum_k \left(\frac{\dot{Q}}{T} \right)_k + \dot{S}_{gen} \left(\frac{kw}{K} \right)$$

C. **Steady state** and **one inlet (1) and one outlet (2) and irreversible** control volume

$$\dot{m} \Delta s = \dot{m}(s_2 - s_1) = \sum_k \left(\frac{\dot{Q}}{T} \right)_k + \dot{S}_{gen} \left(\frac{kw}{K} \right)$$

D. **Steady-state**, one inlet and outlet and **adiabatic** control volume

$$\dot{m}(s_2 - s_1) = \dot{S}_{gen} \left(\frac{kw}{K} \right)$$

E. **Steady state**, one inlet and outlet and **reversible** control volume

$$\dot{m}(s_2 - s_1) = \sum_k \left(\frac{\dot{Q}}{T} \right)_k \left(\frac{kw}{K} \right)$$

F. **Steady-state**, one inlet and outlet **adiabatic and reversible** control volume

$$\dot{m}(s_2 - s_1) = 0 \left(\frac{kw}{K} \right)$$

G. **Steady-state cycle** comprised of open system (control volume) processes

$$0 = \sum_k \left(\frac{\dot{Q}}{T} \right)_k + \dot{S}_{gen} \left(\frac{kw}{K} \right)$$

Can substitute specific properties (kJ/kg-K) for each term above (divide by mass or mass flow rate)

CALCULATING ΔS (KJ/KG-K)

A. **Ideal (incompressible) liquids and solids:** $s_2 - s_1 = C_v \ln\left(\frac{T_2}{T_1}\right)$ where T in Kelvin and $C_v = C_p$

NOTE: isothermal processes in ideal liquids or solids are isentropic.

B. **Ideal gases:** $s_2 - s_1 = C_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right)$ OR $s_2 - s_1 = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)$

Equivalent formulas for ideal gases where T in Kelvin and $C_p = C_v + R$

Problem solving hint: can use isentropic ideal gas process formula to find T given P or v values to then use in 1st law calculation of energy terms.

C. **Non-ideal pure substances use tables for water/steam (A-4 – A-6) or R-134a (A-11 – A-13)**

(i) $s_{\text{mixture}} = x(s_{fg}) + s_f$ (at appropriate T_{sat} or P_{sat})

(ii) $s_{\text{compressed liquid}} \approx s_f$ at system $T = T_{\text{sat}}$

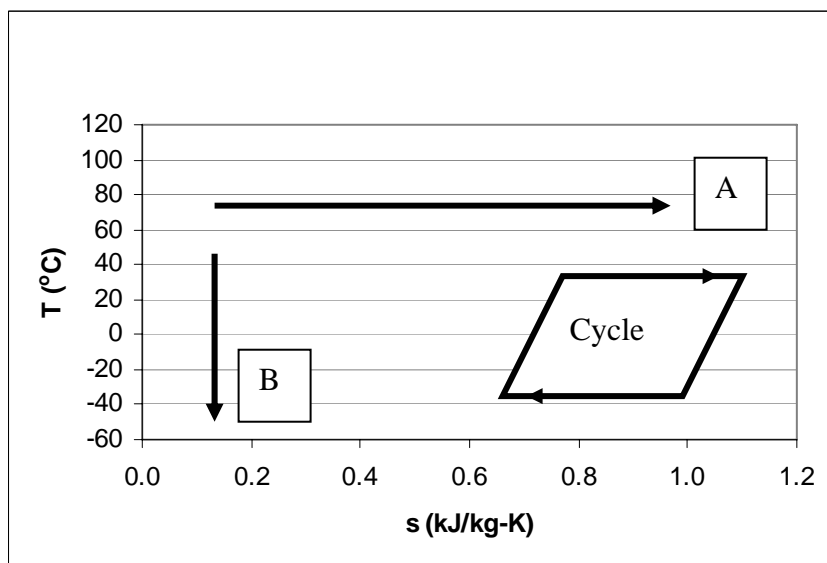
Multiply by mass or mass flow rate for extensive property, kJ/K or kw/K

Entropy Balance Problem solving.

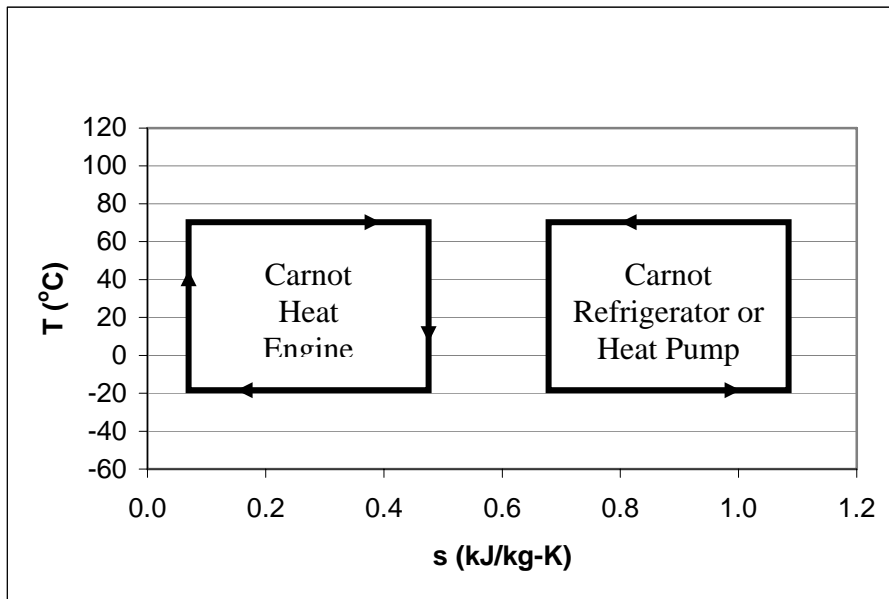
1. Find appropriate entropy balance formula to account for process conditions (e.g., reversible, adiabatic, etc.)
2. Calculate heat (Q) term using the first law, if necessary
3. Calculate change in entropy property term (Δs) using appropriate formula or table

T-S DIAGRAMS

General. For internally reversible processes, area under process line = Q (+ for increasing entropy and – for decreasing entropy during process). Line A is isothermal process. Line B is isentropic process. For Cycle, area enclosed by lines = net heat transfer and $Q_{\text{net}} = W_{\text{net}}$ (1st Law)



T-s diagram for Carnot Cycles consisting of two isothermal processes (one expansion and one compression) and two isentropic processes (one compression and one expansion)



Heat engine: signs for $Q_{net} = W_{net}$ terms will be positive

Refrigerator/Heat Pump: signs for $Q_{net} = W_{net}$ terms will be negative

HEAT ENGINES AND REFRIGERATION CYCLES

I. Heat Engines: Produce work from heat in cycle with some heat rejected.

A. All Heat Engines

$$\eta = \frac{W_{net}}{Q_{net}} = \frac{w_{net}}{q_H} = \frac{\dot{W}_{net}}{\dot{Q}_H} = 1 - \frac{Q_L}{Q_H} = 1 - \frac{q_L}{q_H} = 1 - \frac{\dot{Q}_L}{\dot{Q}_H} = \frac{\dot{W}_{net}}{\dot{m}(q_H)}$$

B. Carnot Heat Engines

$$\eta_{carnot} = 1 - \frac{T_L \text{ (Kelvin)}}{T_H \text{ (Kelvin)}} \text{ and all the relations in part IA.}$$

II. Refrigerators and Heat Pumps: Transfer heat from low temperature reservoir to high temperature reservoir with work input.

A. All refrigerators

$$COP_R = \frac{Q_L}{W_{in}} = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{q_L}{w_{in}} = \frac{1}{\left(\frac{Q_H}{Q_L} - 1\right)} = \frac{1}{\left(\frac{\dot{Q}_H}{\dot{Q}_L} - 1\right)} = \frac{1}{\left(\frac{q_H}{q_L} - 1\right)} = \frac{\dot{m}q_L}{\dot{W}_{in}}$$

B. Carnot refrigerator

$$COP_{R,Carnot} = \frac{1}{\left(\frac{T_H}{T_L} - 1\right)} \text{ and all the relations in part IIA}$$

C. All heat pumps

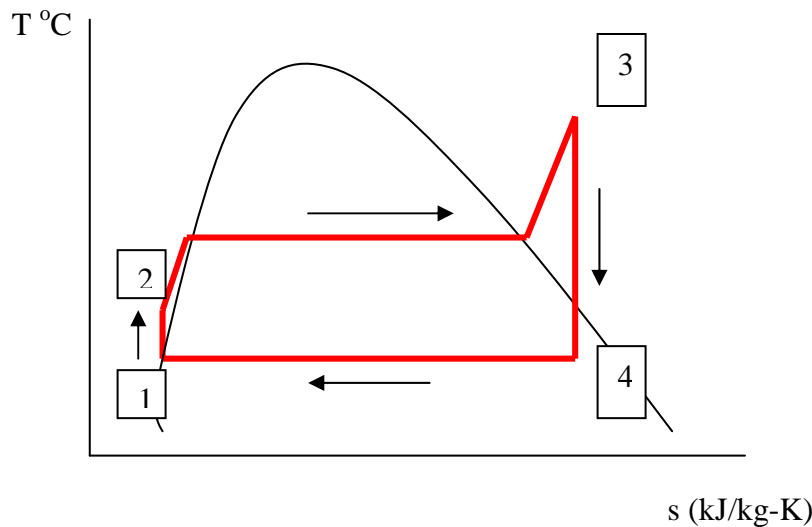
$$COP_{HP} = \frac{Q_H}{W_{in}} = \frac{\dot{Q}_H}{\dot{W}_{in}} = \frac{q_H}{w_{in}} = \frac{1}{\left(1 - \frac{Q_L}{Q_H}\right)} = \frac{1}{\left(1 - \frac{\dot{Q}_L}{\dot{Q}_H}\right)} = \frac{1}{\left(1 - \frac{q_L}{q_H}\right)} = \frac{\dot{m}q_H}{\dot{W}_{in}} = COP_R + 1$$

D. Carnot heat pump

$$COP_{HP,Carnot} = \frac{1}{\left(1 - \frac{T_L}{T_H}\right)} \text{ and all the relations in part IIC}$$

IDEAL RANKINE CYCLE (internally reversible processes okay to evaluate on T-s diagram)

1. Two isentropic processes (pump and turbine)
2. One Isobaric process (boiler)
3. One isothermal and isobaric process (condenser)
4. Typically, unless otherwise specified, saturated liquid at pump inlet, compressed liquid at boiler inlet, superheated vapor at turbine inlet, mixture at condenser inlet.



Rankine Cycle Characteristics

Process	Device	Condition	Energy
1→2	Pump	Isentropic	Work Input, w_P
2→3	Boiler	Isobaric	Heat Input, q_H
3→4	Turbine	Isentropic	Work Output, w_T
4→1	Condenser	Isothermal and Isobaric	Heat Output, q_L

State	Phase	T (°C)	P (kPa)	H (kJ/kg)	s (kJ/kg-K)
1	Saturated liquid	$T_1 = T_{sat} @ P_1$	$P_1 = P_{condenser}$	$h_f @ P_1$	$s_1 = s_f @ P_1$
2	Compressed Liquid	$\sim T_1$	$P_2 = P_{boiler}$	$h_2 = h_1 + v (P_2 - P_1)$	$s_2 = s_1$
3	Superheated Vapor	T_3	$P_3 = P_2$	h_3	s_3
4	Saturated Mixture	$T_4 = T_{sat} @ P_4$	$P_4 = P_1$	$h_4 = x_4(h_{fg}) + h_f$	$s_4 = s_3$

Typically, the pressures for the boiler and condenser and the temperature at the turbine inlet are given. Quality of mixture is found from isentropic turbine, given inlet temperature.

1st Law statements

$$-w_P = h_2 - h_1$$

$$q_H = h_3 - h_2$$

$$-w_T = h_4 - h_3$$

$$q_L = h_1 - h_4$$

$$q_{NET} = w_{NET} \text{ (with appropriate sign conventions)}$$

$$q_H + q_L = w_P + w_T \text{ (with appropriate sign conventions)}$$

Efficiency (efficiency < 1 and efficiency < η_{carnot})

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_H} = 1 - \frac{q_L}{q_H} = 1 - \frac{(h_4 - h_1)}{(h_3 - h_2)} = \frac{\dot{W}_{net}}{\dot{m}(h_3 - h_2)} = \frac{\dot{m}[(h_3 - h_4) - (h_2 - h_1)]}{\dot{Q}_H}$$

Entropy (entropy generated in surroundings > 0)

$$s_{gen} = -\sum_k \left(\frac{q}{T} \right)_k = -\frac{q_H}{T_H} - \frac{q_L}{T_L} = -\frac{(h_3 - h_2)}{T_H} - \frac{(h_1 - h_4)}{T_L} \left(\frac{kJ}{kg - K} \right)$$

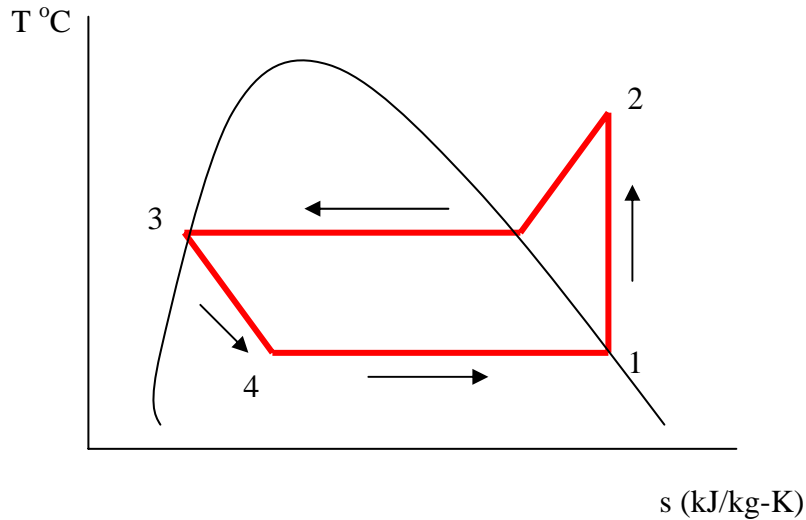
and

$$\dot{S}_{gen} = \dot{m}s_{gen} \left(\frac{kw}{K} \right)$$

given temperature of high- and low-temperature reservoirs (heat source and sink)

IDEAL VAPOR-COMPRESSION REFRIGERATION (VCR) CYCLE (internally reversible processes except for throttle, still okay to evaluate on T-s diagram)

1. One isentropic process (compressor)
2. One isobaric process (condenser)
3. One isothermal and isobaric process (evaporator)
4. One isenthalpic process (throttling valve)
5. Typically, unless otherwise specified, saturated vapor at compressor inlet, superheated vapor at condenser inlet, saturated liquid at throttling valve inlet, saturated mixture at evaporator inlet.



VCR Cycle Characteristics

Process	Device	Condition	Energy
1→2	Compressor	Isentropic	Work Input, w_C
2→3	Condenser	Isobaric	Heat Rejected, q_H
3→4	Throttling Valve	Isenthalpic	--
4→1	Evaporator	Isothermal and Isobaric	Heat Input, q_L

State	Phase	T (°C)	P (kPa)	h (kJ/kg)	s (kJ/kg-K)
1	Saturated vapor	$T_1 = T_{sat} @ P_1$	$P_1 = P_{evaporator}$	$h_g @ P_1$	$s_1 = s_g @ T_{sat}$
2	Superheated vapor	T_2	$P_2 = P_{condenser}$	h_2	$s_2 = s_1$
3	Saturated liquid	$T_{sat} @ P_3$	$P_3 = P_2$	$h_f @ P_3$	$s_f @ P_3$
4	Saturated Mixture	$T_4 = T_{sat} @ P_4$	$P_4 = P_1$	$h_4 = h_3$	$s_4 = x_4(s_{fg}) + s_f @ P_4$

Typically, the pressures for the boiler and condenser are given. T_2 and h_2 are found from isentropic compressor relation and interpolation in table to match $s_2 = s_1$.

1st Law statements

$$-w_C = h_2 - h_1$$

$$q_H = h_3 - h_2$$

$$h_4 = h_3$$

$$q_L = h_1 - h_4$$

$$q_{NET} = w_{NET} \text{ (with appropriate sign conventions)}$$

$$q_H + q_L = w_C \text{ (with appropriate sign conventions)}$$

COP_R

$$COP_R = \frac{\dot{Q}_L}{\dot{W}_C} = \frac{q_L}{w_C} = \frac{(h_1 - h_4)}{(h_2 - h_1)} = \frac{1}{\left(\frac{q_H}{q_L} - 1\right)} = \frac{1}{\left(\frac{(h_2 - h_3)}{(h_1 - h_4)} - 1\right)} = \frac{\dot{m}(h_1 - h_4)}{\dot{W}_C}$$

COP_{HP}

$$COP_{HP} = \frac{\dot{Q}_H}{\dot{W}_C} = \frac{q_H}{w_C} = \frac{(h_2 - h_3)}{(h_2 - h_1)} = \frac{1}{\left(1 - \frac{q_L}{q_H}\right)} = \frac{1}{\left(1 - \frac{(h_1 - h_4)}{(h_2 - h_3)}\right)} = \frac{\dot{m}(h_2 - h_3)}{\dot{W}_C}$$

Entropy (entropy generated in surroundings > 0) for both VCR refrigerator and heat pump

$$s_{gen} = -\sum_k \left(\frac{q}{T}\right)_k = -\frac{q_H}{T_H} - \frac{q_L}{T_L} = -\frac{(h_3 - h_2)}{T_H} - \frac{(h_1 - h_4)}{T_L} \left(\frac{kJ}{kg - K}\right)$$

and

$$\dot{S}_{gen} = \dot{m}s_{gen} \left(\frac{kw}{K}\right)$$

given temperature of high- and low-temperature reservoirs (heat source and sink)