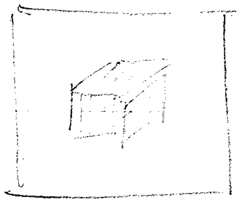
HW 10 SOLUTIONS

1.



0.01 kg chips $C_p = 0.3\ \text{kJ/kgK}$ $T_0 = 20°C$

0.005 kg R-134a $T_1 = -40°C$ $x = 0$

$$\Delta S_{chip} = S_2 - S_0 = m_c C_p \ln\left(\frac{T_2}{T_1}\right)$$

$$\Delta S_{R\text{-}134a} = S_2 - S_1 = m_R (s_2 - s_f), \quad s_1 = 0 \text{ (ref)}$$

Solution: <u>system is adiabatic (assumed)</u>

Assume $T_2 = -40°C$ and test to see if 1st & 2nd Law satisfied

1st Law: $Q = 0$, $W = 0$ for system

$$0 = m_c C (T_2 - T_0) + m_R (u_2 - u_1)$$

$$0 = 0.01\text{kg}\ 0.3\frac{\text{kJ}}{\text{kgK}}(-40 - 20) = 0.005\text{kg}\left(u_2 - (-0.036)\right)\frac{\text{kJ}}{\text{kg}}$$

$$u_2 = 35.96\ \text{kJ/kg}$$

$$x_2 = \frac{35.96 - (-0.036)}{207.4} = 0.174$$

$$m_{g,R} = 0.174(0.005)\text{kg} = 0.00087\ \text{kg}$$

so heat transfer ends when chips are @ $-40°C$ and 17.4% R-134a is vapor and $T_2 = -40°C$

$$\Delta S_{sys} = \Delta S_c + \Delta S_R = 0.01\text{kg}\ 0.3\frac{\text{kJ}}{\text{kgK}} \ln\left(\frac{233}{293}\right) - 0.005\text{kg}(s_2 - 0)$$

$$s_2 = 0.174(0.96856) + 0 = 0.16853$$

1. $\Delta S_{sys} = 0.01(0.3)\left(\ln\left(\frac{233}{293}\right)\right) + 0.005(0.16853)$

$\Delta S_{sys} = -0.00069 + 0.00084$

a) $\Delta S_C = -0.00069 \frac{kJ}{K}$

b) $\Delta S_R = +0.00084\ kJ/K$

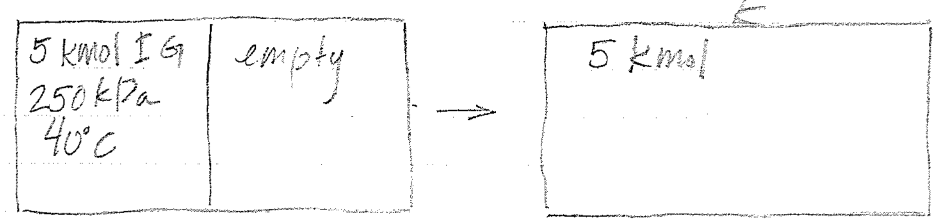
c) $\Delta S_{sys} = 0.00016\ kJ/K$

d) process is possible since

$S_{gen} = \Delta S_{sys} - \frac{Q}{T}$ ($Q \to 0$)

$= 0.00016 \frac{kJ}{K} > 0$

2.



adiabatic

a) $\Delta S = m\left[C_v \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{v_2}{v_1}\right)\right]$

$\frac{v_2}{v_1} = 2$

process is adiabatic + passive

1st Law $0 = mC_v(T_2 - T_1)$

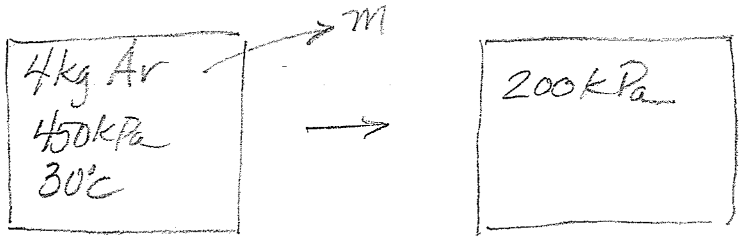
$T_2 = T_1 = 40°C$ isothermal

$\Delta S = mR\ln\left(\frac{v_2}{v_1}\right)$ but $mR = N\bar{R}$

$\Delta S = N\bar{R}\ln\left(\frac{v_2}{v_1}\right) = 8.314\frac{kJ}{kmol\,K}(5\,kmol)\ln(2)$

$\Delta S = 28.81 \frac{kJ}{K}$

3.



adiabatic
reversible

$$S_2 - S_1 = 0 = m\left[C_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right)\right]$$

$$\ln\left(\frac{T_2}{303}\right) = \frac{0.2081}{0.5203}\ln\left(\frac{200}{450}\right) = -0.3243$$

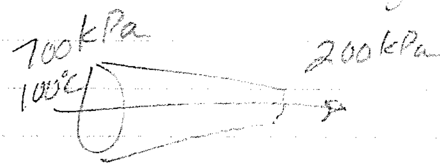
$$T_2 = 219\text{K}$$

$$V = \frac{m_1 R T_1}{P_1} = \frac{4\text{kg}\left(0.2081\frac{\text{kJ}}{\text{kgK}}\right)(303\text{K})}{450\text{kPa}}$$

$$V = 0.56\text{m}^3$$

$$m_2 = \frac{P_2 V}{R T_2} = \frac{200\text{kPa}\,(0.56\text{m}^3)}{0.2081\frac{\text{kJ}}{\text{kgK}}(219\text{K})} = \boxed{2.46\text{kg}}$$

4. $PV^{1.3} = C$



adiabatic

$$P_1 v_1^{1.3} = P_2 v_2^{1.3}$$

$$\left(\frac{v_1}{v_2}\right)^{1.3} = \frac{P_2}{P_1}$$

$$1.3\ln\left(\frac{v_1}{v_2}\right) = \ln\left(\frac{P_2}{P_1}\right)$$

$$\frac{v_1}{v_2} = \exp\left(\frac{\ln(P_2/P_1)}{1.3}\right) = 0.381$$

$$\frac{P_1 v_1}{T_1} = \frac{P_2 v_2}{T_2}$$

a) $$T_2 = T_1\frac{P_2 v_2}{P_1 v_1} = 373\text{K}\left(\frac{200}{700}\right)\left(\frac{1}{0.381}\right) = \boxed{279.35\text{K} = 6.35\,°\text{C}}$$

4. 1^{st} law: $0 = c_p(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2000}$ (4)

$$0 = 1.005\,(279.35 - 373)\,\frac{kJ}{kg} + \frac{V_2^2 - 900}{2000}$$

$$\boxed{V_2 = 435\ m/s}$$

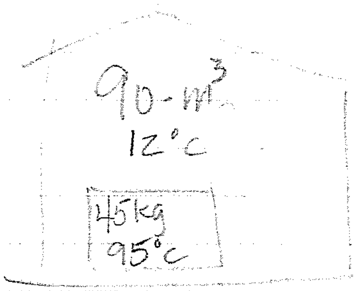
ADD $S_{gen} = c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right)$

$$= 1.005 \ln\left(\frac{279.35}{373}\right) - 0.287 \ln\left(\frac{200}{700}\right)$$

$$-0.2906\ \frac{kJ}{kgK} \quad - \quad (-0.360)$$

$$S_{gen} = 0.069\ \frac{kJ}{kgK}$$ possible irreversible

5.



adiabatic

1^{st} Law, $W = 0$, $q = 0$

a) $0 = m_W\, 4.184\frac{kJ}{kgK}(T_2 - T_1) + \frac{P_1 V_1}{RT_1}\left(c_v (T_2 - T_0)\right)$

$$45\,kg\left(4.184\frac{kJ}{kgK}\right)(95 - T_2) = \frac{101\,(90)}{0.287\,(285k)}\left(0.718\,(T_2 - 12)\right)$$

$$17{,}886.6 - 188.28\,T_2 = 79.8\,T_2 - 957.5\ kJ$$

$$T_2 = \boxed{70.3\ °C}$$

b) $Q = m_w C(T_2 - T_1) = 45\,(4.184)(70.3 - 95)$

$$Q = -4652\ kJ \quad \text{(from water to air)}$$

$$S_{gen} = m_w C \ln\left(\frac{343.3k}{368\,k}\right) + m_a c_v \ln\frac{343.3k}{285k} + R\ln\left(\frac{V_2}{V_1}\right)$$ 0

$$= 45\left(4.184 \ln\left(\frac{343.3}{368}\right)\right) + 111.13kg\,(0.718)\ln\left(\frac{343.3}{285}\right)$$

5. $S_{gen} = -13.08 \frac{kJ}{K} + 14.85 \frac{kJ}{K} = \boxed{1.77 \frac{kJ}{K}}$

(easier to note $V_2 = V_1$ since house is sealed, otherwise you would have to calculate $P_2 = P_1 \frac{T_2}{T_1}$ to solve for S_{gen}

(same answer since $P_2 = 101 \left(\frac{343.3}{285}\right) = 122$ kPa)

6. highest pressure = least work input = isentropic (isothermal and adiabatic)

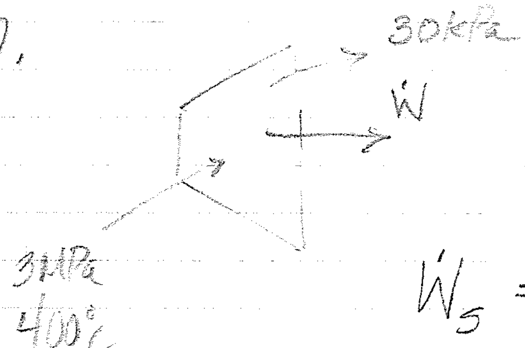
$\dot{Q} = 0$

$$-\dot{W} = v(P_2 - P_1)\dot{m}$$

$$P_2 = \frac{-(-25\,kW)}{5\frac{kg}{s}\left(0.001\frac{m^3}{kg}\right)} + 100\,kPa$$

$$P_{2\,max} = \boxed{5{,}100\,kPa}$$

7.



$\dot{m}_{steam} = 2$ kg/s

$\dot{W}_a = 0.92\,\dot{W}_s$

$\dot{W}_s = \dot{m}(h_2 - h_1)$ for $s_2 = s_1$

$h_1 = 3231.7$ kJ/kg

$s_1 = 6.9235$ kJ/kgK

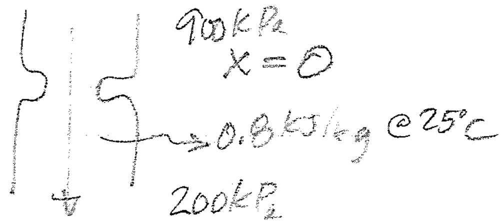
$$X_2 = \frac{6.9235 - 0.9441}{6.8234} = 0.876$$

$h_2 = 0.876(2335.3) + 289.27 = 2335.7$ kJ/kg

$\dot{W}_s = 2\,kg/s\,(3231.7 - 2335.7) = 1{,}792$ kW

7. $\dot{W}_a = 0.92(1792)\,kW = \boxed{1649\,kW}$

8.



1st Law $-0.8\,\frac{kJ}{kg} = h_2 - h_1$

$h_1 = h_f @ 900\,kPa = 101.61\,kJ/kg$

$h_2 = q + h_1 = -0.8 + 101.61$

$h_2 = 100.81\,kJ/kg$

$h_f < h_2 < h_g @ 200\,kPa \Rightarrow$ mixture

$T = T_{sat} = -10.09\,°C$

$s_1 = s_f @ 900\,kPa = 0.37377\,\frac{kJ}{kg\,K}$

$x_2 = \frac{100.81 - 38.43}{206.03} = 0.303$

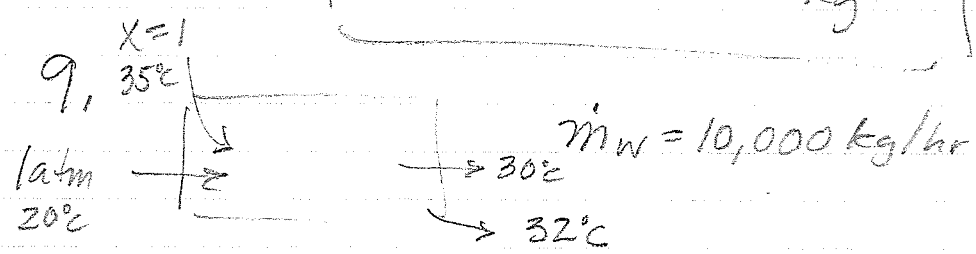
$s_2 = 0.303(0.78316) + 0.15457 = 0.3919\,\frac{kJ}{kg\,K}$

$S_{gen} = s_2 - s_1 - \frac{q}{T} = 0.3919 - 0.37377 - \left(\frac{-0.8}{298K}\right)$

$= 0.0181 + 0.00268$

$\boxed{S_{gen} = 0.0208\,\frac{kJ}{kg\,K}}$

9.



9. $\dot{S}_{gen} = \dot{m}_w \Delta s_w + \dot{m}_a \Delta s_a$ since $\dot{Q} = 0$

$\dot{m}_a$ from 1st Law $\quad h_1 = h_g @ 35°C = 2564.6$

$h_2 \simeq h_f @ 32°C$

$0 = \dot{m}_w (h_2 - h_1)$

$\frac{32-30}{5} = \frac{h_2 - 125.74}{146.64 - 125.74}$

$\dot{m}_a = \frac{\left(\frac{10000}{3600}\right)(2564.6 - 134.3)}{1.005(10)} = 671.7 \, kg/s$ $\quad h_2 = 134.3 \, kJ/kg$

$\dot{m}_w \Delta s_w = \frac{10000}{3600}(s_2 - s_1)$ $\quad s_1 = s_g @ 35°C = 8.3517 \, kJ/kgK$

$s_2 \simeq s_f @ 32°C$

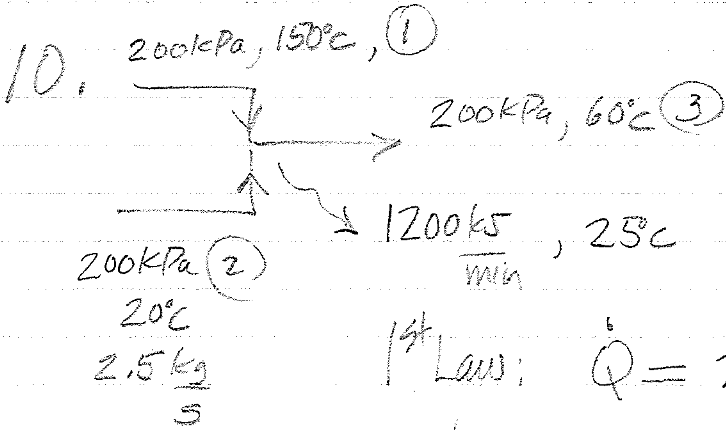
$0.4 = \frac{s_2 - 0.4368}{0.5051 - 0.4368}$, $\; s_2 = 0.46412 \, kJ/kgK$

$= \frac{10000}{3600}(0.46412 - 8.3517) = -21.91 \, kW/K$

$\dot{m}_a \Delta s_a = 671.7(1.005) \ln\left(\frac{303}{293}\right) = 22.66 \, kW/K$

$\dot{S}_{gen} = 22.66 - 21.91 = \boxed{0.745 \, kW/K}$

10.



1st Law: $\dot{Q} = \dot{m}_3 h_3 - \dot{m}_2 h_2 - \dot{m}_1 h_1$

Continuity $\quad \dot{m}_1 + \dot{m}_2 = \dot{m}_3$

$-20 kW = (\dot{m}_1 + \dot{m}_2) h_3 - \dot{m}_1 h_1 - \dot{m}_2 h_2$

$-20 kW = (\dot{m}_1 + 2.5) h_{f,60°} - \dot{m}_1 (2769.1) - 2.5 (h_{f,20°})$

10.

$$-20\,kW = (\dot m_1 + 2.5)\,251.18 - \dot m_1 (2769.1) - 2.5(83.915)$$

$$-438.16 = 251.18\,\dot m_1 - 2769.1\,\dot m_1$$

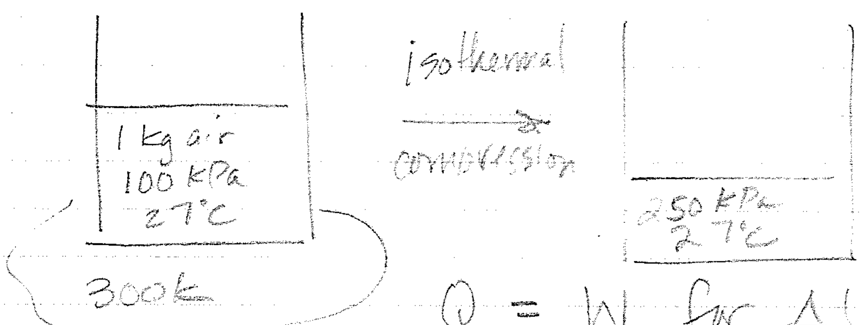
a) $\dot m_1 = \boxed{0.174\ kg/s}$

b) $\dot S_{gen} = \dot m_3 s_{f,60} - \dot m_1 s_{150} - \dot m_2 s_{f,20} - \left(\frac{-20\,kW}{298\,K}\right)$

$$\dot S_{gen} = (2.5 + 0.174)\,\frac{kg}{s}\,0.8313\,\frac{kJ}{kg\,K} - 0.174(7.281) - 2.5(0.2965) + 0.0671$$

$$\dot S_{gen} = \boxed{0.282\ \frac{kW}{K}}$$ irreversible

11.



$$P_1 V_1 = P_2 V_2$$

$$\frac{V_2}{V_1} = \frac{P_1}{P_2}$$

$Q = W$ for $\Delta U = 0$

$$Q = mRT_1 \ln\left(\frac{P_1}{P_2}\right) = 1\,kg\ 0.287\,\frac{kJ}{kg\,K}\ 300\,K\ \ln\left(\frac{100}{250}\right)$$

$$Q = -78.8926\ kJ$$

$$\Delta S_{air} = -mR\ln\left(\frac{P_2}{P_1}\right) = (-1)0.287\ln\left(\frac{250}{100}\right)$$

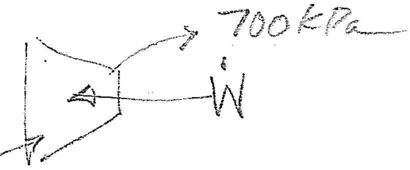
$$\Delta S_{air} = -0.263\ kJ/K$$

$$S_{gen} = -0.263 - \left(\frac{-78.8926}{300\,K}\right) = -0.263 + 0.263$$

$$\boxed{= 0}$$ satisfies 2nd Law

process must be reversible, which makes sense since $T_{res} = T_{sys}$ (no irreversibility)

12.



$\dot{m} = 5\ kg/min$

find $\dot{W}_{min}$ for

a) adiabatic & reversible for min

2nd Law: $s_2 - s_1 = 0 = \dot{m}\left[c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right)\right]$

$$\ln\left(\frac{T_2}{T_1}\right) = \frac{R}{c_p}\ln\left(\frac{P_2}{P_1}\right) = \frac{0.287}{1.005}\ln\left(\frac{700}{100}\right) = 0.556$$

$$T_2 = 290k\,(\exp 0.556) = 505.5k$$

1st Law: $-\dot{W} = \dot{m}(h_2 - h_1) = \dot{m}\,c_p(T_2 - T_1)$

$$= \frac{5}{60}\frac{kg}{s}\left(1.005\frac{kJ}{kg\,k}\right)(505.5 - 290)k$$

$$\boxed{-\dot{W}_{min,\,adia} = 18.0\ kW}$$

b) isothermal reversible for min

1st Law $\dot{Q} = \dot{W}$ since $\Delta T = 0$

2nd Law: $\dot{m}\Delta s_{air} = \frac{\dot{Q}}{T} + \dot{S}_{gen}^{\nearrow 0}$

$$\dot{Q} = 290k\left(\frac{5}{60}\frac{kg}{s}\right)\left[-R\ln\left(\frac{P_2}{P_1}\right)\right]$$

$$\dot{Q} = 290k\left(\frac{5}{60}\frac{kg}{s}\right)\left[-0.287\frac{kJ}{kg\,k}\ln\left(\frac{700}{100}\right)\right]$$

$$= -13.5\ kW = \dot{W}$$

$$\boxed{-\dot{W}_{min\ isoth} = 13.5\ kW} < -\dot{W}_{min,\,adia}$$