Practice problem solutions for midterm 1

1. 1→2 compression ($m = 0.25$ kg air)

$P_1 = 200\text{kPa}$, $P_2 = 300\text{ kPa}$, $V_1 = 0.25\text{m}^3$, $V_2 = 0.1\text{m}^3$

2→3 $V_3 = V_2 = 0.1\text{m}^3$, $P_3 = 100\text{kPa}$ (isochoric)

3→1 $P_3 = P_1 = 200\text{ kPa}$, $V_3 = V_1 = 0.25\text{ m}^3$

a) see graph

b) $T_1 = \frac{P_1 V_1}{mR} = \frac{200\text{kPa}(0.25\text{ m}^3)}{0.25\text{kg}(0.287\frac{\text{kJ}}{\text{kgK}})} = 697\text{K} = 424°\text{C}$

c) without using K or Ap, geometric solution

$A_{12} = W_{12} = \left(\frac{P_1 + P_2}{2}\right)(V_2 - V_1) = \boxed{-37.5\text{ kJ}}$

d) $A_{31} = W_{31} = \left(\frac{P_3 + P_1}{2}\right)(V_1 - V_3) = \left(\frac{100+200}{2}\right)(0.25-0.1) =$

$\boxed{W_{31} = 22.5\text{ kJ}}$

e) $W_{23} = 0$,

$W_{net} = W_{12} + W_{31} = -37.5 + 22.5$

$= \boxed{-15\text{ kJ}}$

f. $W_{S,12} = (300-200)\left(-\frac{0.15}{2}\right) = \boxed{-7.5\text{ kJ}}$

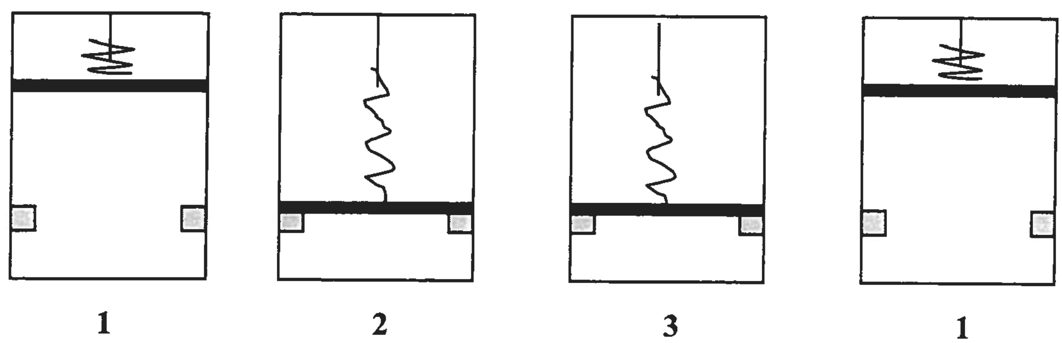
$W_{S31} = (200-100)\left(\frac{0.15}{2}\right) = \boxed{+7.5\text{ kJ}}$ same magnitude, opposite sign

g) $P_1 A + KX = P_2 A$

$X = \frac{A(P_2 - P_1)}{k} = \frac{0.75\text{m}^2(300-200\text{kPa})}{375\text{ kN/m}} = \boxed{0.2\text{ m}}$

AREN 2110
Practice Problems: State, Properties, and Work

1. A device consisting of a piston-cylinder with an attached spring containing 0.25 kg air is operated in the cycle described below

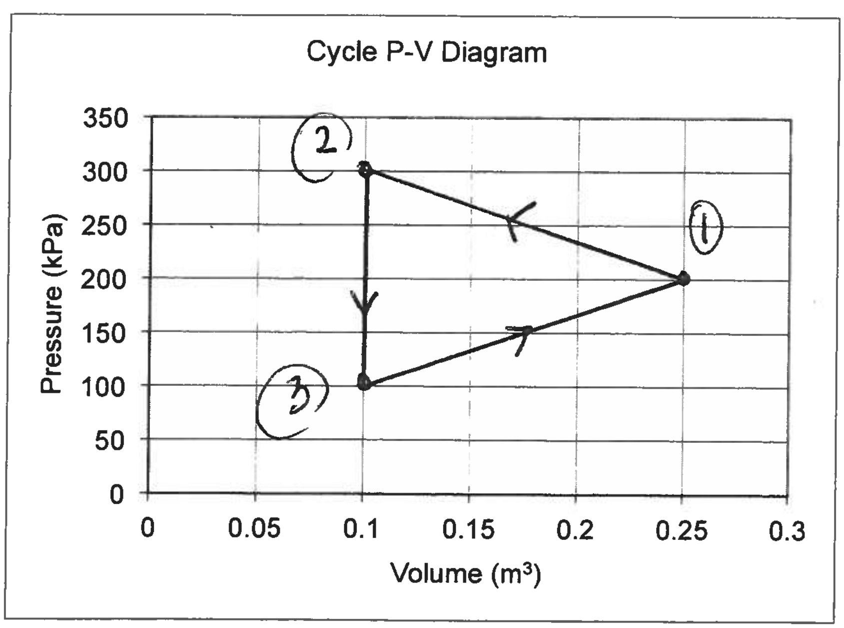


1 → 2: At state 1 $P_1 = 200$ kPa and $V_1 = 0.25$ m^3. The compressed linear spring is released and the air is compressed until the spring no longer stores energy and the piston is just resting on the stops: $P_2 =$ 300 kPa, , $V_2 = 0.10$ m^3

2 → 3: After compression, heat is removed and the pressure drops to 100 kPa at state 3.

3 → 1: Then heat is added, the linear spring is compressed, and the air expands until it reaches state 1.

a) Draw the process on the P-V diagram below.



b) Find the temperature of the air at states 1 and 3 (T_1 and T_3).

c) Calculate the total work done during process 1 → 2.

d) Calculate the total work done during process 3 → 1.

e) Calculate the net work for the cycle, and note if work is being done by the system or on the system.

f) Calculate the work done by the spring in process 1 → 2. How does it compare with the work done on the spring in process 3 → 1?

g) If the spring constant = 375 kN/m and the area of the piston is 0.75 m^2, what is the displacement of the spring for process 1 → 2?

2. Complete the following table

	substance	P (kPa)	T (°C)	v (m^3/kg)	x*	phase
(1)	H_2O	200	120.21	0.3550	0.4	sat. liq-vapor mixture
(2)	H_2O	200	85	0.001032	na	Compressed liquid
(3)	air	200	354	0.9	na	ideal gas
(4)	R-134a	200	-10.09	0.0500	0.50	sat liq-vapor mixture
(5)	R-134a	200	0	0.10481	na	superheated vapor

* use "na" for "not applicable" where quality does not apply

(1) $v = x v_{fg} + v_f = 0.4(0.88578 - 0.001061) + 0.001061 = 0.3550\ m^3/kg$ (A-5)

(2) $v \cong v_f$ @ 85°C (A-4) $T < T_{sat}$ @ 200kPa

(3) $T = Pv/R = 200\,kPa\,(0.9\ m^3/kg)/0.287\,kJ/kgK = 627K = 354°C$

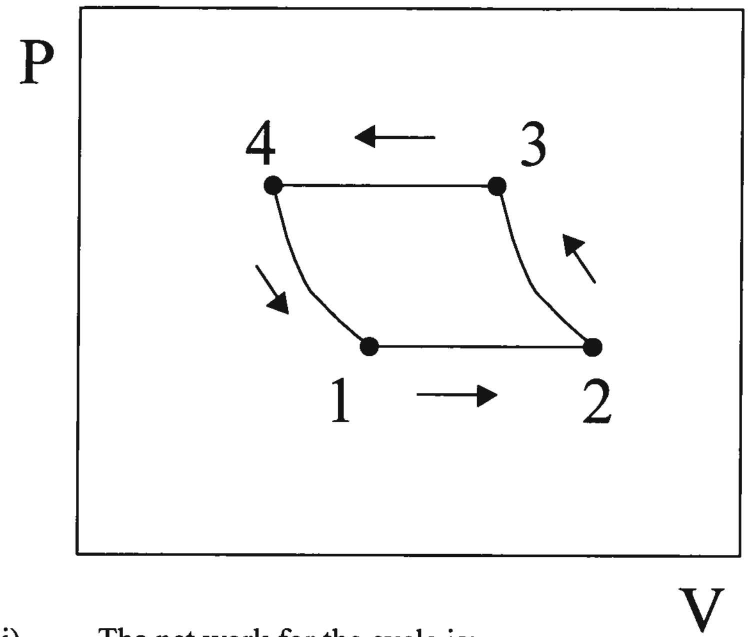
(4) $v_f < v < v_g$ @ 200kPa (A-12), $x = v - v_f / v_{fg}$, $T = T_{sat}$

$x = 0.05 - 0.0007533/(0.099867 - 0.0007533) = 0.50$

(5) $T > T_{sat}$ @ 200kPa (A-12) superheated

A-13 $v = 0.10481$

3. Use the P-V diagram below to answer the following questions



i) The net work for the cycle is:

a) Zero

b) Positive

c) Negative

d) Cannot tell from the diagram

ii) The processes from states 1 → 2 and 3 → 4 are:

a) Isothermal

b) Isobaric

c) Isochoric

d) Isometric

iii) The net enthalpy change for the cycle is

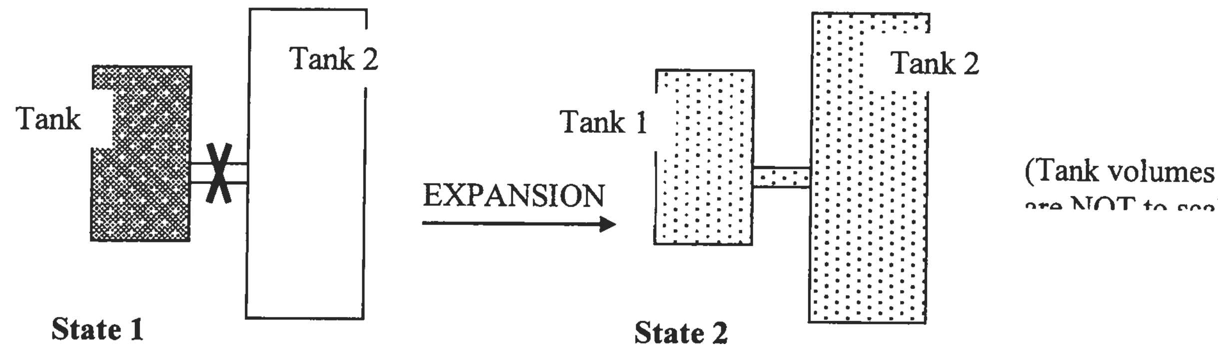
a) Zero

b) Positive

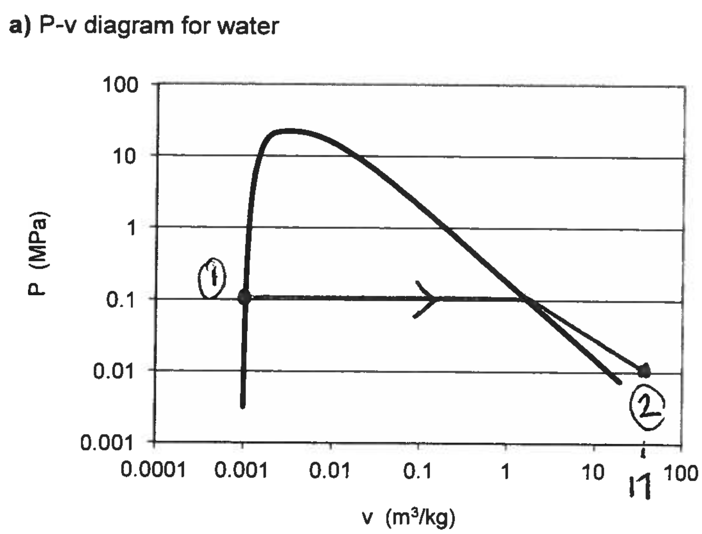
c) Negative

d) Cannot tell from the diagram

4. Ten (10) kg of saturated liquid water at 100 °C and 0.10142 MPa with a specific volume of 0.001043 m^3/kg is contained in a tank connected to an evacuated second tank through a pipe with a closed valve at state 1. The valve is opened and the water expands isothermally into the second tank until equilibrium is reached with 10 kg water at 100 °C and 0.01 MPa at state 2.



a) Draw the process on the P-v diagram below, showing process direction:



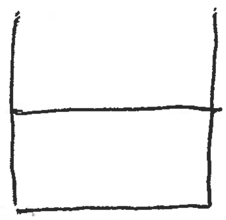
state 1:
sat. liquid water
@ 100°C, P=0.101 MPa
state 2
T=100°C, P_2=0.01 MPa
$P_2 < P_{sat}$ @ 100°C, superheated
$v_2 = 17.196$ m^3/kg
(A-6)

b) Calculate the total volume of the two-tank system.

$$V_2 = m v_2 = 10kg\left(17.196\frac{m^3}{kg}\right) = \boxed{172\ m^3}$$

(6)

5.



H_2O, $m = 1 kg$

$V_1 = 0.2 m^3$

$P_1 = 300 kPa$

$v_1 = \frac{0.2 m^3}{1 kg} = 0.2 m^3/kg$

a) $v_f < v < v_g$ @ 300kPa, sat. mixture $T = T_{sat} =$ **133.5 °C**

b) $v_2 = 0.6058 m^3 = v_g$ @ 300kPa (isobaric expansion)

heat required $= m(h_2 - h_1) = m(h_g - h_1)$

$h_1 = x h_{fg} + h_f$ @ 300kPa

$x = \frac{v_2 - v_f}{v_{fg}} = \frac{0.2 - 0.001073}{0.60582 - 0.001073} = 0.329$

$h_1 = 0.329(2,163.5) + 561.43 = 1,273.1 \ kJ/kg$

$h_g = 2724.9 kJ/kg$

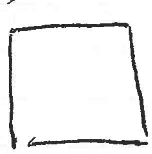
heat required $= 1kg(2724.9 - 1273.1) =$ **1,452 kJ**

c) T_{sat} @ 300kPa = **133.5°C**

d) A-6 **T = 300°C**

e) see graph

6.



H_2O, $m = 2kg$, $V = 0.2 m^3$, $T = 200°C$

$v = \frac{0.2}{2} = 0.1 \ m^3/kg$ $\quad v_f < v < v_g$ mixture

a) $P = P_{sat} =$ **1554.9 kPa**

b) $x = 0.1 - 0.001157/(0.12721 - 0.001157) = 0.784$

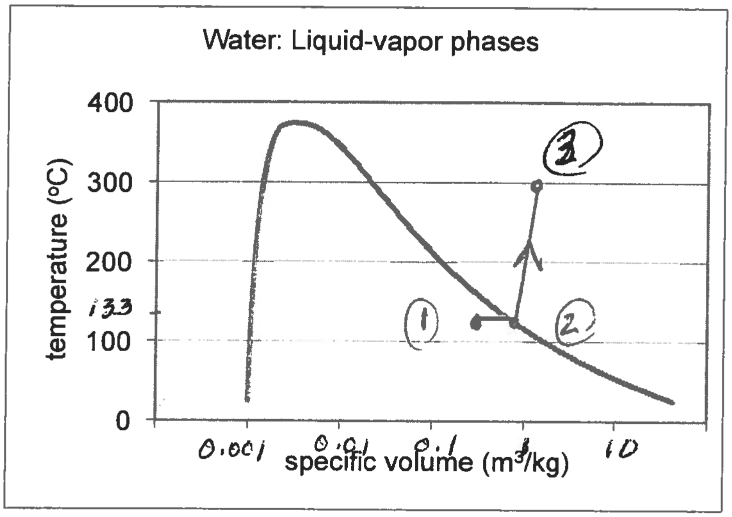
$h = x h_{fg} + h_f = 0.784(1939.8) + 852.26 =$ **2373.3 kJ/kg**

c) $m_g = x m = 0.784(2) kg =$ **1.57 kg** vapor (sat)

d) $V_g = m_g v_g = 1.57 kg (0.12721 \ m^3/kg) =$ **0.199 m³**

5, One (1) kg of water in a piston cylinder device has an initial volume = 0.2 m^3, and pressure of 300 kPa (constant throughout process).

a) What is the water temperature?
b) How much heat is required to expand the water to 0.6058 m^3?
c) What is the temperature after expansion?
d) More heat is added until the volume = 0.8753 m^3. What is the final temperature?
e) Draw the process on a T-v diagram, showing values of for T and v.

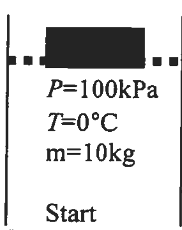


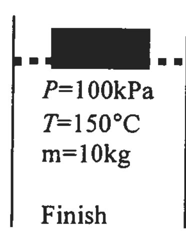
$v_1 = 0.2\ m^3/kg$
$v_2 = 0.6058\ m^3/kg\ (v_g)$
$v_3 = 0.8753\ m^3/kg$
$T_1 = T_2 = 133.5°C$
$T_3 = 300°C$

6. The temperature of two kilograms of water contained in an 0.20-m^3 rigid tank is 200 °C. Determine:

a) the pressure in the system
b) the specific enthalpy of the system
c) the mass of the vapor phase
d) the volume of the vapor phase

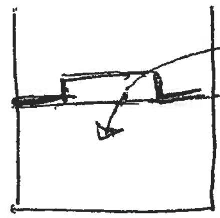
7. A piston/cylinder contains 10 kg of ice at T=0 °C and P=100 kPa. The ice is melted and then warmed to 150°C at constant pressure. (Latent heat of fusion of ice=333.7 kJ/kg @ 0 °C).





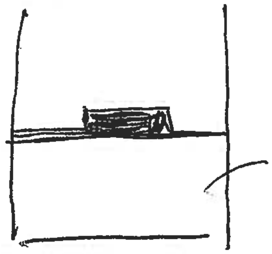
a) How much energy is required to convert the ice to a saturated liquid?
b) How much energy is required to convert the saturated liquid to the final state?
c) What phase is the water at the end of the process?

7.



10kg ice, $T_i = 0°C$, $P = 100 kPa$

$h_i = 333.7 \, kJ/kg$ @ 0°C



10 kg, $T = 150°C$, $P = 100kPa$

a) energy required to melt ice and raise T from 0°C to T_{sat} @ 100kPa, $x = 0$

$$energy = \Delta H = 10kg\left(h_i + (h_f - h_1)\right)$$

$h_1 \cong h_f$ @ 0°C = "0" (ref for table A.4)

h_f @ 100kPa = 417.51 kJ/kg

$$\Delta H = 10(333.7 + 417.51) = \boxed{7,512 \, kJ}$$

b) state 2 $P_2 = 100kPa$, $T_2 = 150°C$ > T_{sat}

superheated vapor at end

8. convert Rankine to Kelvin

$491.7R = 273 K$

a) $\boxed{1 K_{degree} = 1.8 R_{degree}}$ OR

b) BP H_2O @ 1atm = $273K \left(\frac{1.8 \, °R}{K}\right) = \boxed{672 R}$

9. He, isothermal compression $T = 100°C$

$V_1 = 20 m^3$, $V_2 = 2 m^3$; $P_1 = 100 kPa$

a) $m = \frac{P_1 V_1}{R T_1} = \frac{100 kPa (20 m^3)}{2.0769 \frac{kJ}{kgK} (373 K)} = \boxed{2.58 kg}$

b) $W_b = P_1 V_1 \ln\left(\frac{V_2}{V_1}\right) = 100 kPa (20 m^3) \ln\left(\frac{2}{20}\right) = \boxed{-4605 kJ}$

c) isobaric expansion $V_3 = V_1 = 20 m^3$

$P_3 = P_2 = \frac{P_1 V_1}{V_2} = \frac{100 kPa (20 m^3)}{2 m^3} = 1000 kPa$

$T_3 = \frac{P_3 V_3}{mR} = \frac{1000 kPa (20 m^3)}{2.58 kg (2.0769 kJ/kgK)} = \boxed{3732 K}$

d) $W_{net} = W_{b,12} + W_{b23}$

$= -4605 kJ + 1000 kPa (20-2) m^3$

$W_{net} = \boxed{13,395 kJ}$

e) See graph

10. H_2 $m = 3 kg$ $R = 4.124 kJ/kgK$

$C_p = C_p @ 600K = 14.546 kJ/kgK$

energy (Q) $= m C_p (T_2 - T_1) = 3 kg (14.546 \frac{kJ}{kgK}) 400K$

$= \boxed{17,455 kJ}$

$P = \frac{m R T_1}{V_1} = \frac{3 kg (4.124 kJ/kgK) 400K}{10 m^3} = \boxed{495 kPa}$

8. The Rankine scale is an absolute temperature scale. The ice point for water on the Rankine scale = 491.7 R.

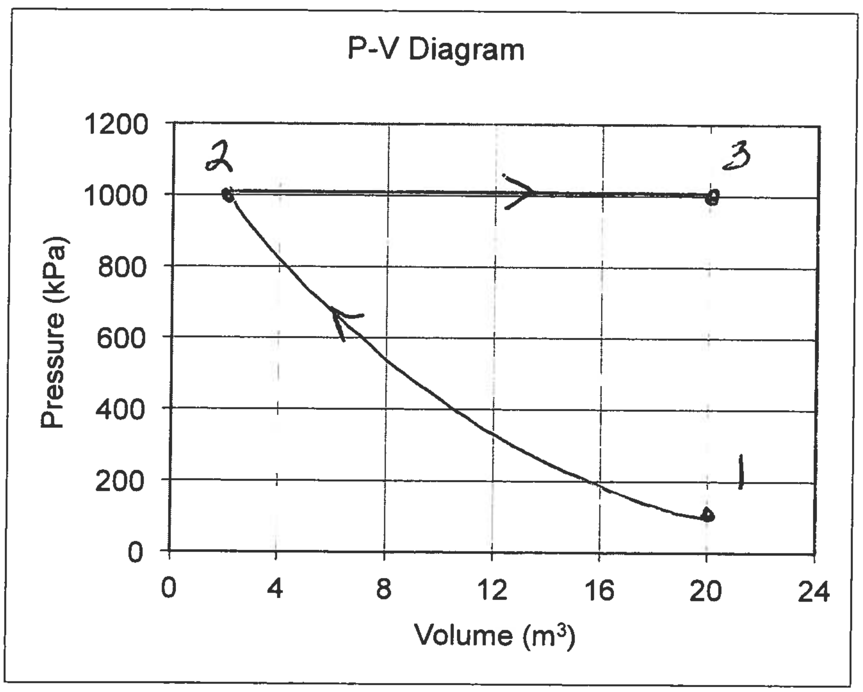
a) Calculate the factor for converting from temperature in the Kelvin scale to the Rankine scale.
b) What is the boiling point of water at 1 atm pressure in the Rankine scale?

9. Helium at 100 °C is compressed in a closed-system, isothermal process from an initial volume of 20 m^3 to a final volume of 2 m^3. The initial pressure of the helium is 100 kPa.

a) What is the mass of helium in the system?
b) What is the boundary work done on the helium during the compression?

After compression, the helium is expanded at constant pressure to its initial volume.

c) What is the final temperature of the helium?
d) What is the <u>net</u> work of the entire two-step process?
e) Draw the process on the P-V diagram below.



10. How much total energy is required to raise the temperature of 3 kg of hydrogen (H_2), an ideal gas, from 400K to 800K in a constant pressure process where the volume of the gas increases from 10 m^3 to 20 m^3? (Assume the temperature change is small enough that the specific heat is constant over the process.) At what pressure does this process take place?

11. Isobaric expansion R-134a

$P = 0.1\ MPa,\ T_1 = -32°C,\ V_1 = 0.1\ m^3$

$T_2 = 30°C$

a) $m = \frac{V_1}{v_1}$

$T_1 < T_{sat}$ @ 100 kPa

compressed liquid

$v_1 \simeq v_f$ @ $-32°C = 0.0007172\ \frac{m^3}{kg}$

$m = \frac{0.1\ m^3}{0.0007172\ m^3/kg}$

$m = \boxed{139\ kg}$

b) $30°C > T_{sat}$ @ 100 kPa (superheated)

$v_2 = 0.24233\ m^3/kg$

$V_2 = 139\ kg\ (0.24233\ \frac{m^3}{kg}) = \boxed{33.8\ m^3}$

c) $h_1 = h_f$ @ $-32°C = 10.1\ kJ/kg$

$h_2 = 280.68\ kJ/kg$

$\Delta H = m(h_2 - h_1) = 139\ kg\ (280.68 - 10.1\ \frac{kJ}{kg})$

$\Delta H = \boxed{37,610\ kJ}$

d) isothermal compression, to sat liquid @ 30°C

$P = P_{sat}$ @ $30°C = \boxed{770.64\ kPa}$

e) See graph

f) $h_3 = h_f$ @ $30°C = 93.58\ kJ/kg$

$\Delta H = m(h_3 - h_1) = 139\ kg\ (93.58 - 10.1) = \boxed{11,604\ kJ}$

11. A closed-system piston-cylinder device contains refrigerant (R-134a) at a pressure of 0.1 MPa, temperature of 241K and initial volume of 0.1 m^3. The refrigerant is expanded in an isobaric process until the temperature reaches 303K.

a) What is the mass of refrigerant in the system?

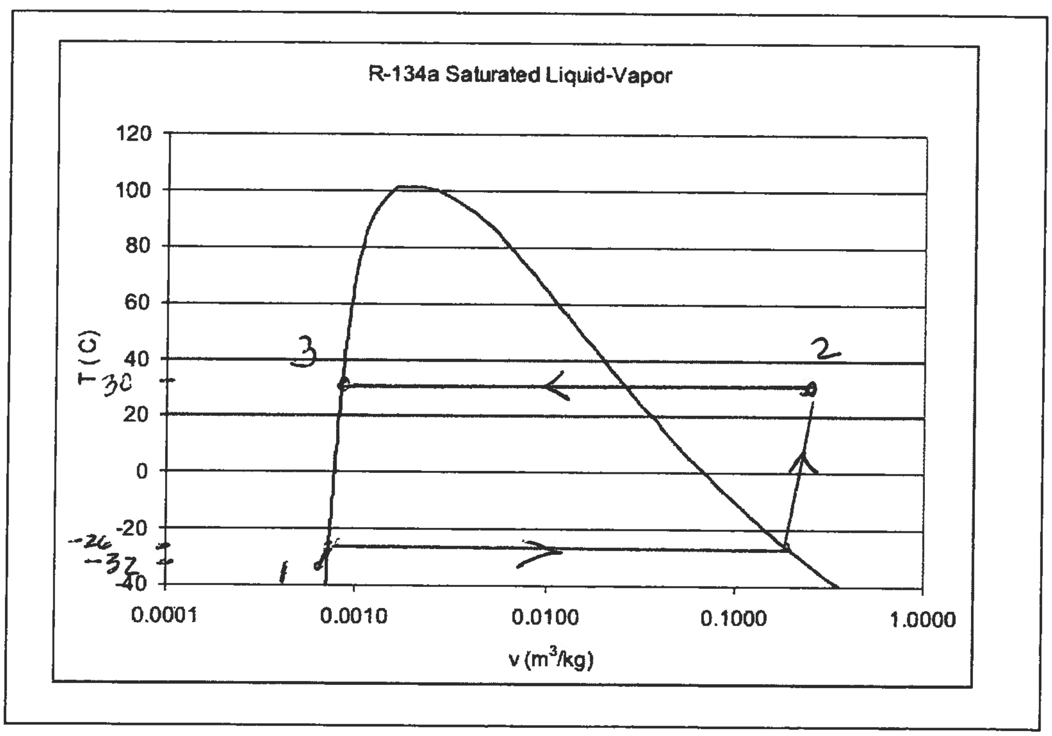
b) What is the total volume of the system after expansion?

c) What is the change in energy of the system during the expansion (in kJ)?

When the refrigerant temperature reaches 303K, energy is removed in an isothermal process until it is a saturated liquid.

d) What is the pressure of the refrigerant at the end of the isothermal process?

e) Draw the two-step process on the T-v diagram for refrigerant below using values for temperature and specific volume:



(13)

f) What is the net change in energy of the refrigerant over the two-step process?

g) Draw the two-step process on the P-v diagram for refrigerant below using values for pressure and specific volume.